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Search Engine Dominance and Quality of Organic Search: Consequences for Online Advertising

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Contents

Abstract

In this dissertation work, we study in which way an Internet search engine can lower the quality (score) of the organic links, and how it will affect the probability of the high quality website to get a high position in the organic search results. We investigate the impact of such organic search results manipulation on advertisers' strategies, search engine's profits, and consumer welfare. We find that when organic link quality becomes too high, a search engine can face the cannibalisation problem when consumers prefer to click on organic links first and consequently satisfy their needs, rather than click on sponsored links – revenue source for the search engine. Thus, a search engine starts reducing organic link quality that reflects in lower consumer surplus. According to the performed analysis, investment in paid placement gives a guarantee that the website will be visited at least on the sponsored site. Modelling the impact of search engine's organic search results manipulation on the advertisers' optimal search engine marketing strategies reveals that, facing the downward pressure on the organic rank, at least one advertiser will invest in paid placement.

Key words

Internet search engine, Search engine optimization (SEO), Paid placement (PP), Ranking algorithm, Cost-per-click (CPC), Ad auction, Organic links, Sponsored links, Organic search, Organic search quality, Search engine marketing (SEM) strategy.

1 Introduction

Nowadays, in the era of internet technologies, everything can be sold and bought via the Internet. Every day billion of users visit search engines typing specific key words, related to the information about products or services that consumers wish to buy. Due to this fact, websites start to compete for their places in the online search to appear on the first pages. Thus, search engine marketing becomes a dominant player in the sphere of online advertising.

To make online marketing to be more efficient, ***online search engines*** like Google, Bing, Yahoo, Yandex, etc. consistently up-grade search results by ranking links according to their relevance level for the search query. Most online search engines have divided their search results into organic and sponsored (paid) parts. Organic search results appear on the left part of the screen, while a paid placement is on the right side. Being displayed in organic search results, an advertiser doesn't pay anything to the search engine (SE). On the contrary, an advertisement for a fee on the basis of a specific auction mechanism is located in the right part of a search screen and calls a sponsored link. In this case, an advertiser pays to a search engine for displaying the ads as the sponsored links. To allocate an ad in Google's sponsored results is rather easy through the ***AdWords account***.

Certainly, investing in a paid placement is a very fast way to attract more consumers, but it is also rather an expensive one. In order to locate ads among the most visible links, advertisers make bids, and usually only the highest bids win. After getting the sponsored links, advertisers start paying for each user's click on the ad (***Pay-Per-Click*** model), regardless whether the user is a random visitor of the website or she is a potential consumer. Since an ad will be displayed as the sponsored link only till the moment, while an advertiser doesn't run out of the budget on the AdWords account (in case of the usage of Google search engine), the effect from such an ad allocation generally is rather temporary. Thus, advertisers have started investing not only in the paid placement, but also in improvement of their organic placement by influencing the search engine's ranking algorithm. The process of such organic search results refinement is called ***Search Engine Optimization (SEO)***.

An investment in SEO has several important advantages. In some cases, SEO can be less expensive and have not only a positive effect in a short run but also in a long run. SEO implies special methods and techniques directing to put a link on the top of organic search results. There two types of SEO exist: ***white hat SEO*** *and* ***black hat SEO***. White hat SEO includes techniques that increase the quality of a website mostly by improving its content and, as a result, increase visitor satisfaction. Widely known white hat SEO techniques are ***SEO copywriting***, ***Meta tags***, etc.

Black hat SEO assumes techniques that refine the ranking of a website without improving its quality. Generally, online search engines' policy is oriented on indexing only quality website's content that increases visitor satisfaction. But, in practice, it is rather difficult to detect the content, written with the usage of either white hat SEO or black hat SEO. Hence, the most frequently used black hat SEO technique is a purchase of ***external links*** to the website, filling the website by not relevant hidden content with a lot of key words, or other programming tricks as ***302 Redirect***, and so on. Since black hat SEO aims to manipulate the ranking process of the search engine without increasing in the relevance of a website, a search engine uses punitive sanctions when websites conducting black hat SEO are caught. In an extreme situation, a search engine can ban the website and remove it from the organic search results.

Nevertheless, if search engines could only disapprove black hat SEO around 3 years ago, since 2011 they actively have started making significant investments in black hat SEO reduction trough the increase in their ***algorithm robustness*** (Xing and Lin, 2006). Thus, in February 2011 Google introduced its innovation – ***Panda algorithm***. The algorithm was directed on the ranking in organic search results in a way to lower the position of "bad-quality" websites, which had no relevant and unique content, and reveal black hat SEO practices. Once the algorithm was run, around 12% of all English language websites lost their positions in organic search results. Many top ranked websites got the places much lower than they had before Panda algorithm's start up. During the last year the algorithm was being repeatedly updated. On April 24, 2012 it was introduced ***Google Penguin*** that affected next 3.1% of all English language websites[1].

To explain its position, Google, as well as other search engines, points out that the usage of SEO in order to manipulate a ranking process not only hurts consumer satisfaction, but also decreases the benefits of "transparent" sites. In addition, search engines claim that their auction mechanism for sponsored links allocation guarantees top links to be of the highest quality that assures mutual welfare. But, following this logic, it is also possible to state that SEO mechanism is similar: the richest sites significantly invest in SEO and get top positions in organic search results, as well as the richest sites with the highest bids get their top positions in paid placement. Hence, it is much likely that an advertiser, investing in SEO more than others, will have better quality of the content. Thus, it is rather interesting why search engines are against SEO and don't allow sites to improve their positions in organic search results using this method?

The answer is rather obvious. Since income, obtained from the sponsored links allocation, is the main revenue source of a search engine, it becomes clear why SEs are much encouraged in SEO efficiency decreasing. More advertisers decide to invest in SEO rather than in paid placement, less revenue is got by the search engine (Berman and Katona, 2012).

[1] See "Another step to reward high-quality sites" and "More guidance on building high-quality sites". Official Google Webmaster Central Blog.

In the conditions of fair competition, a search engine with the highest quality results will get maximum revenue. However, contradicting this logic, since organic and sponsored listings are competitors within one search engine, it can be the case that a SE would have an incentive to move away higher quality sites from the top organic results (Taylor, 2013). Certainly, such practice is detrimental for social and consumer welfare. In the case when a search engine has monopolistic power and, therefore, is not afraid to loose its consumers and advertisers, the situation can become dramatic. Having unrestricted power, a search engine can manipulate the organic search results through the ranking algorithm in order to increase its revenue.

Evidently, possessing a dominant market position, a company can use a wide range of manipulation anti-competitive tools to get extra profits. Search engines have power not only to manipulate search results in their own commercial interests, but to use other methods to get the largest market share running out competitors and taking the opportunity to get the whole ‘pie’.

In spite of the many-sided issue of the search engine’s abuse of its dominant position, our main research interest is focused on the problem of the SE’s manipulation of organic search results and its impact on all participants, such as a SE itself, advertisers and consumers.

Due to the relatively recent rise of interest towards this issue, rather small literature was devoted to the problem of the SE’s abuse of its dominance. Thus, our main aim is to fill the gap in the existed literature and to analyze this issue in our research study.

Therefore, our research question is: ***What is the role of the dominant search engine in online advertising?***

In order to answer the research question, the following objectives are set:

1. To explain SEO process, its interaction with sponsored links, and its impact on a SE.
2. To determine conditions when a SE has an incentive to manipulate the ranking within search results.
3. To study an effect of the SE’s manipulation of organic search results on SE itself, consumers and advertisers.

The thesis is structured as follows. In the first chapter we review recent literature concerning search marketing and online advertising. Thus, we analyze the paper of Ron Berman and Zsolt Katona (2012) “The role of search engine optimization in search marketing”, which shows how search engine optimization can affect the probability of a high quality website or a low quality website to get the first position in organic search results, and affect advertiser’s and search engine’s profits; the paper of Greg Taylor (2013) “Search quality and revenue cannibalization by competing search engines”, which gives an answer on the question ‘when an internet search engine can have an incentive to degrade its organic search results quality’. In the second chapter we set up the model that allows studying the impact of the search engine’s manipulation of its organic search results on the SE profits, advertisers’ strategies and consumer welfare.

2 Literature review

Apart from the plenty of literature, devoted to the analysis of the sponsored lists (Athey and Ellison, 2011; Edelman, Ostrovsky and Schwarz, 2007; Varian, 2007), there exist several research studies that describe not only the process of paid search, but also the process of the organic listing and its interplay with the paid one. Thus, Katona and Sarvary (2010), White (2009), Hu, Chen and Whinston (2008), Xing and Lin (2006), Etro (2012), Berman and Katona (2012) and Taylor (2013) propose models that assume a search engine's provision of both sponsored and organic links.

Primary focus in the analysis of Katona and Sarvary (2010) is the interaction between two processes: a search engine's provision of the organic search results according to the specific search context and selling of sponsored links to generate revenues. In this study, advertisers compete for the sponsored links where the intensity of their competition depends on the advertisers' positions in the organic listing and the behaviour of consumers in online search. Katona and Sarvary study a dynamic model where advertisers bid each period for the sponsored links and consumers' clicks have a lagged effect that is explained by the consumers' memory of the website's address they regularly visit. Due to the lagged effect, advertisers can loose an incentive to compete for the sponsored links next period. However, such lagged effect has a tendency to decrease over time. Furthermore, contrary to the existing literature, the authors determine the SE's endogenous choice of the number of offered sponsored links, which depends on the amount of traffic devoted to each link.

The main results of the authors' analysis show that, depending on the different attractiveness levels of the websites, the advertisers' benefits from the sponsored links can also be different. Thus, if the websites are attractive and have the high rank in the organic listing, advertisers benefit less purchasing paid links. Moreover, if consumers don't trust the websites with sponsored links, advertisers can loose clicks not only on the sponsored side, but also on the organic one. Consequently, less attractive advertisers can end up with the higher probability to win the sponsored auction. However, if the level of the websites' attractiveness is very high, the advertiser will inherently get much traffic and will be a winner of the sponsored auction.

In turn, the paper of White (2013) sheds light on how a search engine benefits from the provision of high quality organic ("unpaid") search results together with the paid advertisements. Organic links are selected by the SE's ranking algorithm, whereas sponsored links are displayed according to the advertisers' payments. The revenues of the SE depend directly on SE's ability to "give market power" to advertisers through the selling of sponsored links. The SE is interested to keep the quality of the organic search results on the high level since, by doing so, it gives larger market power to advertisers.

White (2013) describes the relationship between the search engine's choice of the organic search results quality and a fee for the sponsored links. If the quality of the organic listing is high, the SE can charge higher price for paid placement. Since, investing in sponsored links, advertisers compete for the market positions, it is ambiguous whether an increase in the advertising fee causes a raise of the price of the products they sell. The author shows that when high organic search quality benefits all the consumers equally, advertisers will increase their prices. However, when the improvement in quality of organic listing benefits only newbies, an advertiser will keep her price on the lower level. In addition, providing high quality organic links, the SE makes advertisers, who purchase paid placement, to compete not only with each other, but also with the websites on the organic side. Such competition is especially tough when websites on the organic side and on the sponsored side sell similar products. Certainly, it exerts downward pressure on the market price of the products advertisers charge.

Nevertheless, it becomes clear that search engine's profits strongly depend on the quality of the organic search results, White claims that this issue has to be studied more carefully and should be based on more progressive models that could allow analysing the relationship between the SE's choice of the organic listing quality and the auction mechanism to sell the ad slots.

The study of Xing and Lin (2006) is quite different from Katona's and Sarvary's (2010) or White's (2013) ones. Xing and Lin (2006) are ones of the fewest authors who analyse not only paid placement, as online marketing strategy on the advertising market, but also search engine optimization. Xing and Lin define the conditions when SEO exists, and study the impact of SEO on the advertising market. Contrary to Sen (2005), who analyse choices of advertisers' online marketing strategies, such as paid placements, SEO or no strategy, the work of Xing and Lin focuses on the advertisers' net benefit from the investment in SEO. Similar study has been conducted by the followers of Xing and Lin, Berman and Katona (2012).

The novelty of Xing's and Lin's work is their new approach to the definition of search engine quality. Since existing literature associates the search engine quality only with the user satisfaction and doesn't anyhow determine the impact of SEO on such quality, Xing and Lin make an attempt to define two dimensions of search engine quality, such as "algorithm efficiency" and "algorithm robustness". Ranking algorithm efficiency and ranking algorithm robustness determine a search engine's ability to accurately rank relevant websites in the organic search.

Having high efficiency of the ranking algorithm, a SE is able to satisfy all the consumers who search on it. In turn, algorithm robustness corresponds to a SE's ability to deter black hat SEO and punish advertisers who violate search engine SEO guidelines. The authors claim that, if algorithm robustness is high, SEO (black hat SEO) is more expensive and less likely to be implemented.

The results of the analysis show that, being investing in algorithm robustness; a SE protects its algorithm efficiency. Thus, it gives the practical insight for a search engine about its profitability in the conditions of SEO implementation.

While highlighting the process of a search engine's provision of both sponsored and organic search results, the existing literature generally doesn't address the question whether and when a dominant search engine has an incentive to manipulate its search results.

The study of Etro (2012) is the one that sheds light on this issue and gives both qualitative and quantitative justification of SE's manipulation process. Thus, Etro claims that, in order to make an interaction between consumers and ads more likely to occur, the search engine aims to increase artificially the quality of its organic links, which lead to SE's own vertical search services. Such anti-competitive behaviour is primarily directed to encourage consumers' usage of the vertical search services, proposed by a search engine, and divert traffic from the competitors. When consumers' cost to compare and evaluate the quality of competing search engines is rather high, the search engine can get much more traffic due to its possibility to bias proposed search results.

The results of the analysis show that the dominant search engine always has an incentive to increase the degree of manipulation and divert traffic from the competing services because the potential loss of visitors is fully compensated by the higher return on investment.

Certainly, in this case, consumer and social benefits are lower: users are not fully satisfied by visiting websites of not really high quality, and advertisers are more likely forced to pay higher price for an ad allocation due to SE‘s monopolistic power growth.

2.1 The role of search engine optimization in search marketing

Berman and Katona are ones of the pioneers in the study of the role of search engine optimization (SEO) in search marketing and online advertising, and ones of few authors who analyze organic search. Berman and Katona study the impact of the SEO on the profits of advertisers, who compete for top organic and sponsored links. The main result of their analysis shows that a positive level of search engine optimization can refine the search engine's organic ranking quality and increase the satisfaction of the visitors.

When a search engine doesn't provide sponsored links, an organic ranking can be improved by SEO if and only if there is a positive correlation between the quality of the website content and consumers' valuation (the website's traffic). Moreover, given that paid search results are available, the positive effect of SEO on the organic ranking quality becomes even stronger and holds regardless of the correlation. It happens when the opportunity to invest in sponsored links is the second chance for a website to get more visitors' clicks. Thus, a website with low quality content doesn't choose to invest substantially in SEO and gives this advantage to its high quality counterparts. Since consumers expect the organic side of online search results to be of high quality, they start their search by clicking organic links first.

Hence, Berman and Katona find that, in spite of the positive influence of SEO on the consumer welfare and the surplus of a high quality advertiser, search engine's revenues are less when advertisers substitute their investment in the sponsored links by the investments in SEO. Authors show that the relationship between a minimum bid for a sponsored link (a reserve price), set by the search engine, and SE's profits has an inverse U shape, proposing an optimal minimum bid that is decreasing in the level of SEO activity.

The concept of Berman and Katona (2012) is fundamental in our research study. Since the authors give a hint why a search engine can be uninterested in advertisers' investments in SEO and, instead, would prefer to attract websites to pay for sponsored links, we are based on this intuition also in our dissertation work.

Model

A model is built as a static game, where consumers search for the information entering key words, and advertisers compete for the traffic, which is coming from consumers. It is assumed the existence of the monopolistic search engine. The main role of the search engine is to provide organic search results and sponsored links according to the scoring mechanism (in organic search) or advertisers' bids, corrected on the click-through rates (for paid placement). Main characteristics and incentives of the search engine, consumers and advertisers in this static game are explained further.

Websites and Consumers

In the model there are two players (websites). Each website provides one unit of a good that has quality $q_i \in \{q_L, q_H\}$, with q_H (high quality) > q_L (low quality). The good is assumed to be information, content or a physical product. Net price of a good is the consumer utility of q_i. q_H and q_L are common knowledge, but consumers need to find out particular qualities of goods, provided by each website. To discover quality of the good, provided by the website, consumers click on organic or sponsored links, proposed by the search engine. While searching on a website, visitors incur cost, $c \geq 0$, directed on observing the quality. Once a good is found, consumers take the decisions, whether to buy it or not, or to continue the search. Decisions, concerning which link (organic or sponsored) to click first, depend on the expected quality distribution for each link. A rational consumer will search for a good only till the moment, while expected utility growth from the visiting next link will be higher than the sequential search cost. When a consumer decides to stop search, she will choose among all found goods the one, which brings the highest net utility.

Thus, in the case when a consumer starts her search by clicking on organic links and finds q_H, there will be no reason for her to continue search, and she will consume the good with the net utility $q_H - c$. But, if a consumer finds q_L, instead of q_H, among the organic links, she will continue searching incurring further costs. If a consumer finds again q_L among the sponsored links, she will be charged $2c$ with the decreased net utility $q_L - 2c$.

Selling a good, the website receives exogenously determined revenue (net of manufacturing costs) valued at $v_i \in \{v_L, v_H\}$ with v_H (high valuation) > v_L (low valuation). Total revenue of site *i* can be calculated as the number of consumers, who buy the good, multiplied by v_i (no consumer will execute transaction twice). Each website's quality q_i and valuation v_i are known to competing sites, but are unknown to consumers and to the search engine. However, the distribution of the qualities is the common knowledge:

$$\Pr(q_i = q_L) = \Pr(q_i = q_H) = \frac{1}{2}, \quad \Pr(v_i = v_L) = \Pr(v_i = v_H) = \frac{1}{2}.$$

Thus, the probabilities to be a low type or a high type in quality are equal, as well as the probabilities in valuations. ρ is denoted as the correlation between q_i and v_i for each website *i*, where qualities and valuations are independent across websites. The sign of the correlation between quality and valuation can be determined by the specific factors at the market. Implying positive correlation between quality and valuation (good example is *Amazon.com*), firms, which are operating on the vertically differentiated markets and offer higher quality products, can charge higher price for their products and receive higher profits. Negative correlation between quality and valuation can occur when websites have 'spam' characteristics, providing a low quality good, but getting high traffic.

In order to increase their ranking in organic and paid search, websites have two possibilities. They can invest efforts e_i in SEO, incurring quadratic cost $e_i^2/2$, to succeed in the organic ranking, or/and they can submit per-click bids b_i to place an ad among sponsored links. The total cost for the sponsored link is determined by the ***generalized second price auction*** with the minimum bid r, where bids are corrected by the expected ***click-trough rates*** (CTRs) (Athey and Ellison, 2011**)**. Thus, final payoff of website *i* is calculated as the difference between total revenues and total costs (investments in SEO and paid placement).

The search engine

A search engine is an intermediary that allows meeting consumers and advertisers (websites). While the main goal of a search engine is to get profits through the sale of sponsored links, the task of a SE is to provide the highest quality organic search results. In

order to give the best websites the highest rank positions, a search engine uses ***crawling algorithms*** and data mining methods that allow estimating the quality of a certain website. Since the quality is measured not accurately, the error can not be observed directly.
Authors propose the following model for each website's score estimation:

$$s_i = q_i + \alpha e_i + \varepsilon_i, \tag{2.1.1}$$

where α is the regression coefficient, denoting the effectiveness of SEO (*black SEO*), and ε_i is an error with the distribution c.d.f. F_ε and mean 0. Thus, α reflects the ease to change website's ranking by SEO, which is increasing if a search engine doesn't pay attention on SEO activity.

As it was mentioned above, a search engine provides sponsored links according to the click-through rates, which correct bids in the second price auction, with the reserve minimum bid of r. Thus, if website *i* has expected click-trough rate ctr_i, it will get score $ctr_i \cdot b_i$, if the score is higher than r. Each time a consumer clicks on the sponsored link, the website is charged the bid of the next highest bidder, corrected for the click-trough rate differences. Thus, click-through rates determine the profits of the search engine and influence advertisers' incentives to invest in SEO.

Timing

Authors determine the following time points in the game. The game starts with the search engine's publication of the minimum bid for sponsored links, r. Simultaneously, Nature defines q_i and v_i for each website with the correlation coefficient ρ, independent across sites. Then, websites choose how much effort e_i to invest in SEO. After the search engine gives score s_i for each site in the organic ranking, websites bid for the sponsored links that are provided on the basis of CTR-corrected generalized second price auction with the minimum bid r. When both rankings are executed, search process starts.

2.1.1 SEO Equilibrium. Organic links only

In on-line advertising there is a common situation when websites don't possess enough resources to buy sponsored links. It often happens when sites provide free content, and their only revenue source is a paid allocation of the ads *(a search engine pays websites for its ads allocation;* ***AdSense*** *(in case of Google))*. In this situation, revenues of the sites are rather small and the profits are lower than the minimum bid for sponsored links. Thus, such websites prefer to invest in SEO, rather than in paid placement. Hence, the expected payoff of website *i* is

$$\pi_i = v_i \cdot \Pr(s_i > s_j) - \frac{e_i^2}{2}, \qquad (2.1.1.1)$$

where $\Pr(s_i > s_j)$ is the probability that site *i* will get the higher score than site *j*. It is assumed that the measurement error has a uniform distribution $\varepsilon_i \sim U\left[-\frac{\sigma}{2}, \frac{\sigma}{2}\right]$ with the support $\sigma > q_1 - q_2$, for the error to have any effect. To investigate an impact of SEO on the organic ranking and consumer welfare, it is imposed $P(\alpha) = P(\alpha; \sigma, v_1, v_2, q_1, q_2)$, denoted as efficiency of the ranking, which is the probability that the highest quality website will win the organic link. Since the utility of a consumer coincides with the quality of good q_i, consumer welfare increases in the efficiency $P(\alpha)$.

When SEO is not possible, i.e. $\alpha = 0$, $P(0) < 1$ as long as $q_1 \neq q_2$ because of the noise in the ranking process. Moreover, $P(0, \sigma)$ is decreasing in σ, since higher level of the noise makes the ranking less efficient[2]. When SEO is available, i.e. $\alpha > 0$, websites can affect the ranking in their own interests. Proposition 1 summarizes how search engine optimization influences the ranking, consumer's and firm's benefits when sponsored links are not available.

Proposition 1

1. When $\rho = 1$, any $\alpha > 0$, which is not too large, improves the efficiency of the ranking and consumer satisfaction. However, when $\rho = -1$, SEO is harmful for consumer satisfaction. For an intermediate value of the correlation coefficient, $-1 < \rho < 1$, SEO can increase consumer satisfaction for a certain α value.

[2] $P(\alpha)\big|_{\alpha=0} = 1 - \frac{\sigma^2(\sigma-\mu)^2}{2(\sigma^2 + \alpha^2(v_1 - v_2))^2} = 1 - \frac{\sigma^2(\sigma-\mu)^2}{2(\sigma^2)^2} < 1$ (see proof of proposition 1);

$$\frac{\partial P(\alpha,\sigma)\big|_{\alpha=0}}{\partial \sigma} = -\frac{1}{2}\frac{((2\sigma(\sigma-\mu)^2 + 2\sigma^2(\sigma-\mu))(\sigma^2)^2 - 4\sigma^3(\sigma^2(\sigma-\mu)^2)}{(\sigma^2)^4} < 0$$

2. When α is small, and $\rho = -1$, both sites' profits are decreasing in α. When $\rho = 1$, sites' profits are decreasing in α, except for the higher quality site, whose profits are increasing iff $v_H > 2v_L$.

Proof

Let $F_{\varepsilon_i - \varepsilon_j}$ be the c.d.f. of a triangle distribution $\varepsilon_i - \varepsilon_j \sim T[-\sigma, \sigma]$ with a mean zero and $f_{\varepsilon_i - \varepsilon_j}$ be its p.d.f.

From $s_i = q_i + \alpha e_i + \varepsilon_i$, e_i can be expressed as

$e_i = \dfrac{s_i - q_i - \varepsilon_i}{\alpha}$, and its expected value is $E[e_i] = \dfrac{\bar{s}_i - q_i}{\alpha}$, since an error has a mean zero.

Profit of the website is $\pi_i = v_i \cdot \Pr(s_i > s_j) - \dfrac{e_i^2}{2}$.

Expected profit is $\pi_i = v_i \cdot F_{\varepsilon_i - \varepsilon_j}(\bar{s}_i - \bar{s}_j) - \dfrac{1}{2}\left[\dfrac{\bar{s}_i - q_i}{\alpha}\right]^2$.

Maximization of the profit π_i with respect to score $\bar{s}_i$ gives the following F.O.C.:

$$v_i \cdot f_{\varepsilon_i - \varepsilon_j}(\bar{s}_i - \bar{s}_j) = \frac{\bar{s}_i - q_i}{\alpha^2}, \qquad (2.1.1.2)$$

where $\bar{s}_i = E_{\varepsilon_i}[s_i]$. Let $x = \bar{s}_i - \bar{s}_j$ and $\mu = q_i - q_j$. By subtracting both F.O.Cs and using the fact that $f_{\varepsilon_i - \varepsilon_j}$ is symmetric around zero, we can rewrite the conditions as:

$$f_{\varepsilon_i - \varepsilon_j}(x) = \frac{x - \mu}{\alpha^2 (v_i - v_j)}. \qquad (2.1.1.3)$$

An interior solution x^* requires both F.O.Cs and S.O.C.s are satisfied as well as $-\sigma \le x^* \le \sigma$.

Since an error has a triangular distribution, $f_{\varepsilon_i - \varepsilon_j}(x)$ has the following form:

$$f_{\varepsilon_i - \varepsilon_j}(x) = \begin{cases} (\sigma + x)/\sigma^2, & -\sigma \le x \le 0 \\ (\sigma - x)/\sigma^2, & 0 \le x \le \sigma \end{cases}$$

Combining this system of equations with the (2.1.1.3), we get two equilibrium solutions of x^*.

If $0 \le x \le \sigma$, when $v_i > v_j$ and $\alpha^2 \ge -\sigma \frac{\mu}{v_i - v_j}$, *(*$\alpha^2 = -\sigma \frac{\mu}{v_i - v_j}$*, when x=0)* or when $v_i < v_j$ and $\alpha^2 < \sigma \frac{\mu}{v_j - v_i}$, the equilibrium solution is $s_i^* - s_j^* = x_R^* = \frac{\sigma^2 \mu + \sigma\alpha^2 (v_i - v_j)}{\sigma^2 + \alpha^2 (v_i - v_j)}$.

If $-\sigma \le x \le 0$, when $v_i < v_j$ and $\alpha^2 \ge \sigma \frac{\mu}{v_j - v_i}$, or when $v_i > v_j$ and $\alpha^2 < -\sigma \frac{\mu}{v_i - v_j}$, the equilibrium solution is $s_i^* - s_j^* = x_L^* = \frac{\sigma^2 \mu + \sigma\alpha^2 (v_i - v_j)}{\sigma^2 - \alpha^2 (v_i - v_j)}$.

Condition $\sigma > \mu$ ($\sigma > q_i - q_j$ *for the error to have any effect*) ensures $-\sigma \le x \le \sigma$, while $\alpha^2 < \frac{\sigma^2}{v_H}$ ensures both F.O.C. and S.O.Cs hold. Thus, with the condition on α, the equilibrium point is a unique extremum and a global maximum.

In order to check the effect of the equilibrium SEO investment on the ranking efficiency and consumer satisfaction, let $P(\alpha)$ denotes the probability of the winning organic link by the site with the highest quality.

$P(\alpha) = F_{\varepsilon_1 - \varepsilon_2}(x^*)$. Given a triangle distribution, $F_{\varepsilon_1 - \varepsilon_2}(x^*)$ has the following form:

$$F_{\varepsilon_i - \varepsilon_j}(x^*) = \begin{cases} \frac{(x+\sigma)^2}{2\sigma^2}, & -\sigma \le x \le 0 \\ 1 - \frac{(\sigma - x)^2}{2\sigma^2}, & 0 \le x \le \sigma \end{cases}$$

Assuming $q_H = q_1 > q_2 = q_L$, in the perfectly correlated case, when $x^* = x_R^*$,

$$P(\alpha) = F_{\varepsilon_1 - \varepsilon_2}(x_R^*) = 1 - \frac{\sigma^2 (\sigma - \mu)^2}{2(\sigma^2 + \alpha^2 (v_1 - v_2))^2}$$ and

$$P'(\alpha) = f_{\varepsilon_1 - \varepsilon_2}(x_R^*) \frac{\partial x}{\partial \alpha}\bigg|_{x = x_R^*} = \frac{2\alpha\sigma^2 (v_1 - v_2)(\sigma - \mu)^2}{(\sigma^2 + \alpha^2 (v_1 - v_2))^3} > 0.$$

In the perfectly negatively correlated case, when $\rho = -1$, and $x^* = x_L^*$,

$$P'(\alpha) = f_{\varepsilon_1 - \varepsilon_2}(x_L^*) \frac{\partial x}{\partial \alpha}\bigg|_{x = x_L^*} = -\frac{2\alpha\sigma^2 (v_1 - v_2)(\mu + \sigma)^2}{(\sigma^2 - \alpha^2 (v_1 - v_2))^3} < 0.$$

In the conditions of an intermediate correlation it can be shown that $P(\alpha) > P(0)$ for certain $\alpha > 0$ and $0 < \rho < 1$.

Proving the second part of the proposition 1, it can be shown that, when $\rho = -1$, taking the derivative with respect to α of the profit functions of both players, at the limit of $\alpha \to 0$, the profit never increases for any of the equilibrium conditions:

$\left.\frac{\partial \pi_1}{\partial \alpha}\right|_{x_L^*} = -\frac{2v_i \alpha \sigma^2 (v_i - v_j)(\mu + \sigma)^2}{(\sigma^2 - \alpha^2 (v_i - v_j))^3}$ goes to zero as $\alpha \to 0$.

At the same conditions, the profit might increase for the higher values of α. When $\rho = 1$, solving for the player 1:

$$\pi_i(\alpha) - \pi_i(\alpha = 0) = \pi_i(\alpha)\big|_{x^*} - v_1 F_{\varepsilon_1 - \varepsilon_2}(q_H - q_L) > 0 \qquad (2.1.1.4)$$

gives the condition $v_H > 2v_L$ or $\alpha^2 > \frac{\sigma^2 (2v_L - v_H)}{(v_H - v_L)^2}$. The same procedure for player 2 shows that there is no solution for $\alpha > 0$ that increases player 2's profit.

Hence, main results show that equilibrium SEO investments have a positive effect on the ranking mechanism when valuations are positively correlated with qualities. In this case, higher quality sites get an incentive to improve their organic ranking position through the investments in SEO, due to SEO's ability to correct the ranking mechanism error. On the other hand, in case of the negative correlation between valuations and qualities, lower sites ("spammer" sites) could also have a potential incentive to increase their traffic trough the SEO. Certainly, providing non relevant content, such sites would hurt consumers' utility.

Nevertheless, the analysis of this study counter-intuitively shows that low quality sites will never be better off with an increase in SEO effectiveness. Only high quality websites have an advantage to benefit because their valuations are much higher than competitors' ones. Thus, a privilege in higher valuation will help better sites to win the organic link, while low quality websites with high valuations will never benefit from SEO due to the tough competition with high quality sites.

2.1.2 SEO Equilibrium. The role of sponsored links

In this section, it is allowed the availability of buying sponsored links by the websites. It is important to examine how sponsored advertising can affect websites' incentive to invest in SEO. By-turn, it is crucial to analyze the impact of SEO on the search engine's revenues, mainly obtained from the sale of sponsored links. The model is solved using the constraint

$r < v_H$. It means that a site with the high valuation will definitely able to afford a payment for a sponsored link. Despite, describing the intuition, authors focus on the situation when any site can invest in paid placement ($r < v_L$).

In order to understand advertisers' priorities to invest in SEO and paid placement, it is critical to investigate consumer behavior in search process. Since it is assumed that consumers are rational and they incur small search costs, it considers that they will start the search with the link that gives the highest probability to find a higher quality good.

The following results of the analysis are indicated in the proposition 2.

Proposition 2 There exists $\bar{c} > 0$, such that if $c < \bar{c}$ then

1. In the unique equilibrium consumers start the search process on the organic side.
2. If $r < v_L$, the likelihood to win the organic link increases in α for any $-1 < \rho < 1$.
3. If $v_L \le r$, the likelihood to win the organic link increases in α for ρ high enough.
4. SE's revenues are increasing in α iff the likelihood to win the organic link is decreasing.

Proof

In order to prove the results of the Proposition 2, it has been used the backward induction method. At the first stage advertisers determine their bids for the sponsored links, given their positions in organic search results. Then they decide how much to invest in SEO in three different cases, given the qualities of the corresponding websites. From the beginning it was assumed that consumers start their search clicking on the organic links. Later, it will be shown that such strategy is an equilibrium one. In addition, it is determined the threshold clicking costs, $\bar{c}$. The analysis starts when $r < v_L$, and then, extends to the case when $v_L \le r < v_H$. A winner of the organic link is denoted as w_O and a winner of the sponsored link as w_S . The main procedure in this analysis is a comparison between equilibrium profits of a player if she gets or not the organic link. The difference between such profits is denoted as a value of the organic link for the player.

Case 1: If $q_i = q_j = q_H$, consumers will search only on the organic side and will not go to the sponsored one. Thus, players will not pay for sponsored links and will invest only in SEO. This situation is equivalent with the SEO game without sponsored links.

Case 2: If $q_i = q_j = q_L$, consumers will switch on the sponsored links after unsuccessful search on an organic side. They will continue search until a sponsored link doesn't lead to the same site, they found before in the organic search. If site *i* is the organic winner, then it will not pay for sponsored link ($ctr_i = 0$), thus site $j \neq i$ will be a winner of a sponsored link paying minimum price r per click. Since $q_i = q_j$, consumers will not come back to the organic links after finding the same site on a sponsored link, and, as a result, site *i* will not get any profit. Therefore, no site will have an incentive to invest in SEO.

Case 3: If $q_i = q_H$ and $q_j = q_L$, consumers will stop their search on the organic link when the winner of an organic link $w_O = i$. Similarly to the Case 1, no site will pay for sponsored links more than minimum reserve price r. On the contrary, if a winner of an organic link $w_O = j$, consumers will continue the search, clicking on a sponsored link as long as it differs from the organic one, since they have been not satisfied on an organic side before. Thus, $ctr_i = 1$ and $ctr_j = 0$ with price per click r for a winner of a sponsored link, $w_S = i$. Hence, site *i* that has high quality will capture all the demand in the market, even not being a winner of an organic link[3].

If a winner of an organic link is $w_O = i$, site *i* will get profits $\pi_i^O = v_i$, but if $w_O = j$, and a winner of a sponsored link is $w_S = i$, site i pays for the sponsored link and $\pi_i^S = v_i - r$. Thus, the value of winning an organic link is $v_i' = \pi_i^O - \pi_i^S = r$ for site i and $v_j' = \pi_j^O - \pi_j^S = 0$ for site j. Given $v_i' = r, v_j' = 0, q_i' = q_H, q_j' = q_L,$ and applying results got from Proposition 1, optimal SEO investments and the probability to get an organic link by high quality site are:

$$e_i^* = \frac{\alpha r(\sigma - q_H + q_L)}{\alpha^2 r + \sigma^2}, e_j^* = 0,$$

$$P^* = P(\alpha | q_i = q_H, q_j = q_L) = 1 - \frac{1}{2}\left(\frac{\sigma(\sigma - q_H + q_L)}{\alpha^2 r + \sigma^2}\right)^2. \qquad (2.1.2.1)$$

Optimal level of SEO investments has been found in the following way:

[3] The case, when site *i* captures all the market, must assume demand discontinuity (similarly to Bertrand competition case). Consumer loyalty is not allowed in this case.

Given $E[e_i] = \frac{\bar{s}_i - q_i}{\alpha}$ and F.O.C. $v_i' \cdot f_{\varepsilon_i - \varepsilon_j}(\bar{s}_i - \bar{s}_j) = \frac{\bar{s}_i - q_i}{\alpha^2}$, it is possible to express $e_i^* = \alpha v_i' f_{\varepsilon_i - e_j}(\bar{s}_i - \bar{s}_j) = \alpha v_i' f_{e_i - e_j}(x)$.

Since $v_i' = r$, $e_i^* = \alpha r \frac{\sigma - x_R}{\sigma^2} = \alpha r \frac{\sigma - \frac{\sigma^2 \mu + \sigma \alpha^2 (r-0)}{\sigma^2 + \alpha^2 (r-0)}}{\sigma^2} = \frac{\alpha r(\sigma - \mu)}{\alpha^2 r + \sigma^2} = \frac{\alpha r(\sigma - q_H + q_L)}{\alpha^2 r + \sigma^2}$.

Since $v_j' = 0$, $e_j^* = \alpha v_j' f_{\varepsilon_i - e_j}(\bar{s}_i - \bar{s}_j) = 0 \cdot \alpha f_{e_i - e_j}(x) = 0.$

Probability to get an organic link by a high quality site has been obtained in the following way:

$$P^* = P(\alpha | q_i = q_H, q_j = q_L) = F_{\varepsilon_i - \varepsilon_j}(x_R^*) = 1 - \frac{(\sigma - x_R^*)^2}{2\sigma^2} = \frac{2\sigma^2 - \sigma^2(\sigma - \mu)^2 + 2\alpha^2 r}{(2\sigma^2 + 2\alpha^2 r)^2} =$$

$$= 1 - \frac{1}{2}\left(\frac{\sigma(\sigma - q_H + q_L)}{\alpha^2 r + \sigma^2} \right)^2.$$

Since P^* is increasing in α, $w_O = i$ more likely to occur as α increases regardless of ρ, proving second statement of Proposition 2.

When $v_L \le r < v_H$, a winner of an organic link is $w_O = j$ and $v_i = v_L < r$, site i with $q_i = q_H$ doesn't have enough resources to pay for a sponsored link and will have profit $\pi_i^O - \pi_i^S = v_L - 0 = v_L$ from getting an organic link, while site j will get $\pi_j^O - \pi_j^S = v_H - 0 = v_H$. Proposition 1 states that higher α decreases the probability to get an organic link, except the case, when the site has higher quality.

Here, $\Pr(q_i = q_H, q_j = q_L, v_i = v_L, v_j = v_H) = \left(\frac{1-\rho}{4} \right)^2$, that is decreasing in ρ and equal 0 when ρ =1 (when ρ =1, a higher quality website will have also higher valuation). Thus, proving the third statement of Proposition 2, SEO efficiency will increase the probability to get an organic link by the high quality site only if ρ is high enough.

From the above analysis of 3 possible cases, it is clear that an organic link will be given to the high quality site more likely than a sponsored link. Thus, in equilibrium, rational consumers will start their search on the organic side. To determine $\bar{c}$, it is necessary to calculate the expected benefit of continuing the search by a consumer when she finds q_L, that is, using Bayes rule:

$$(q_H - q_L)\Pr(q_{W_S} = q_H \big| q_{W_O} = q_L) = (q_H - q_L)\frac{(1/2)(1-P^*)}{(1/4)+(1/2)(1-P^*)} = (q_H - q_L)\frac{1-P^*}{3/2-P^*},$$

where P^* is defined in (2.1.2.1) and

(1/2)(1 – P^*) = $\Pr(q_{w_o} = q_L \mid q_{w_s} = q_H)\Pr(q_{w_s} = q_H)$. Since to start search, for a consumer it is enough to have $c < q_L$,

$$\bar{c} = \min\left(q_L, (q_H - q_L)\frac{1-P^*}{3/2-P^*}\right). \qquad (2.1.2.2)$$

To prove the fourth statement of Proposition 2, it is enough to examine Case 3 since in first two cases consumer welfare and the search engine's revenues are not affected by SEO. Thus, consumers always look for q_H, but their net utility is higher if search costs are less (and $w_O = i$). Thus, consumers are better iff $P(\alpha)$ increases. Nevertheless, the search engine's revenues are higher if a low quality website gets an organic link, and consumers will click on a sponsored link to find the high quality good. Thus, the search engine's revenues increase iff $P(\alpha)$ decreases.

In sum, it has been proved that the presence of sponsored links creates an additional incentive for advertisers to invest in SEO and get an organic link in order not to pay for a sponsored one. Unless the search engine claims that SEO is harmful since it helps low quality sites to increases their ranks in the organic search, these results show that even if a low quality site becomes a winner of an organic link, consumers will still look for the high quality good and will find it clicking on a sponsored link. Hence, an opportunity of paid placement is the second chance for the high quality site to win a link, sponsored, in this case. But, since SEO is a distinct advantage primary for high quality websites, which have a much larger competitive power, low quality websites lose an incentive to invest in SEO, and, as a result, high quality sites are more likely to win the SEO game.

Since high quality websites try to avoid substantial payments for sponsored links allocation, they start to invest in SEO, diverting traffic from the low quality competitors. In this case, the probability to get an organic link by the high quality site is increasing; and a rational consumer starts her search exactly on an organic side. Therefore, SEO increases the welfare of consumers. But, on the contrary, with the increasing probability to win an organic link, the search engine's revenues decrease since advertisers have much less incentives to pay for sponsored links. Hereby, SEO becomes a big danger for the SE, not in diminishing efficiency of the ranking algorithm, but primary in decreasing of the revenues coming from the sponsored links.

Revenues of the search engine directly depend on minimum bid r. When SEO is absent, revenues are increasing in the minimum bid, whereas if SEO is possible, the situation is different:

Corollary 1 There exists $\hat{r}(\alpha) > 0$ such that the search engine's revenues are increasing in r, for $r < \hat{r}(\alpha)$, and decreasing, for $\hat{r}(\alpha) < r < v_L$. When v_L is high enough, then $\hat{r}(\alpha)$ is the unique optimal minimum bid, which is decreasing in α.

Proof

Consumers click on a sponsored link and the SE gets revenues only if an organic link is of low quality. Since, in this case, the SE gets exactly r, its expected revenues are $R^{SE} = (1 - P(\alpha)) \cdot r$, where $P(\alpha)$ is the probability that the higher quality website wins an organic link. Thus, from the proof of Proposition 1,

$$P(\alpha) = P(\alpha, r) = P(q_{W_O} = q_H) = 1/4 + 1/2P^* = \frac{3}{4} - \frac{1}{4}\left(\frac{\sigma(\sigma - q_H + q_L)}{\alpha^2 r + \sigma^2}\right)^2,$$

which is increasing in r. Differentiating revenues with respect to r gives:

$$\frac{\partial R^{SE}}{\partial r} = 1 - P(\alpha, r) - r \cdot \frac{\partial P(\alpha, r)}{\partial r} = \frac{1}{4} + \frac{\sigma^2(\sigma - q_H + q_L)^2(\sigma^2 - r\alpha^2)}{4(\sigma^2 + r\alpha^2)^3}.$$

The derivative is positive if r is below $\hat{r}(\alpha)$.

The inverse U-shape relation between the minimum bid and the SE's revenues can be explained by two opposite forces. When the SE starts increasing the minimum bid, revenues also increase, but further bid growth pushes advertisers to switch their investment from sponsored links to SEO. Till the definite moment, when advertisers invest much more in SEO than in sponsored links, the SE's revenues start decreasing. Maximal profits are reached by the SE earlier than SEO effectiveness increases.

Corollary 2 If $r < v_L$ and two sites have different qualities, the profit of the higher quality site increases, while the profit of the lower quality site decreases in α.

Proof

When $r < v_L$, the higher quality website has effective valuation $v'_H = r$, and the low quality website has $v'_L = o$. Thus, the higher quality website has higher chances to get an organic link and higher profits as SEO effectiveness increases.

When efficiency of SEO increases, the higher quality website invests less in SEO that increases its profit, whereas the lower quality website will loose profits due to the higher competitive power of the higher quality website. In case when both websites have low qualities, SEO will benefit those one with higher valuation.

Summing up, Berman and Katona show that there exists the possibility to improve the SE's organic ranking quality and increase the satisfaction of consumers with the positive level of SEO (black hat SEO), when website's qualities and valuations (traffic) are positively correlated. Moreover, high quality websites are always better of than low quality websites, and even if they don't get the highest ranking positions in organic search, they have a second chance to win the sponsored links and attract more visitors, than lower quality websites; in equilibrium, consumers always start their search on the organic side; the SE is worse off when SEO efficiency increases.

2.2 Search quality and revenue cannibalisation by competing search engines

Introducing a new product, a company can face the so called *cannibalization* problem. It may happen when two products of one firm have competing market positions, and “a new product gains sales by diverting them from an existing product” (Heskett, 1976). The same scenario is applicable to an internet search engine that provides not only sponsored links, but also organic ones. Therefore, links, displayed on the right and on the left sides of the search screen, become competing “services” within the one search engine.

Obviously, if consumers are used to meet their needs on an organic side, they will not click on sponsored links incurring additional costs (cost of time, for example). Since clicks on organic links, contrary to ones on paid links, don’t generate revenue for a SE, the online search engine has a particular interest to attract visitors on the sponsored side, rather than on the organic one. One way to do it is the manipulation of the organic search results quality. But, even if it seems rather easy to “force” consumers to click on sponsored links by decreasing organic search quality, in reality, a SE can lose its competitiveness due to the failure of offering best possible organic results. Thus, an ambiguity of the consequences from such manipulation doesn’t give an answer, whether a SE indeed has or not an incentive to affect its search results.

In his paper, Taylor (2013) addresses the question “when and why might competing search engines benefit from deliberately degrading their results quality?” Empirical studies show that consumers do use organic links to satisfy their needs. Hence, to attract consumers, a SE needs to provide high quality organic results, especially when switching costs to visit another SE are high. Not satisfying her need on an organic side, a consumer will stay on the SE and continue clicking on paid links. However, providing high quality organic results, a SE will divert traffic from the sites with paid links. Therefore, a SE faces the dual problem, whether to compete for gaining a larger market share by providing higher quality organic search results or to degrade organic search quality by attracting consumers to a paid side.

Despite such a trade-off, Taylor (2013) shows that there exists an equilibrium, in which search engines, even being competitors, deliberately degrade their organic search quality. Nevertheless, when consumers exhibit loyalty, a SE will choose technologically feasible maximal organic search quality that can not rise higher than a placed ceiling.

The logical question, arisen from the possibility of the equilibrium quality degradation, is about consequences for consumer welfare. With the reference to switching costs, one can expect that a reduction in such costs can stimulate the competition between search engines. However, Taylor (2013) claims that SEs will have a small incentive to compete if consumers can easily switch between search engines earlier than they click on paid links. And, vice versa, if switching costs are high enough, search engines will compete more for consumers by sustaining a high level of organic search quality, because consumers will prefer to choose only one search engine to visit, clicking both on its organic links and then on paid ones. Thus, counter intuitively, existence of high switching costs can make consumers better off due to SE's maintaining the high level of organic quality.

The intuition of Taylor's study is based on the empirical analysis of Gandal (2001), who has found that consumers are willing to switch between search engines in order to find what they need. Moreover, Yang and Ghose (2010) have showed that a SE's provision of organic links increases the amount of clicked sponsored links. Therefore, availability of organic search is a powerful tool to attract more consumers.

Hence, in his study, Taylor (2013) assumes that consumers are attracted by high quality search results; there exists not only the competition between search engines, but the cannibalization problem, reflecting in consumers' substitution of paid links by the organic counterparts of high quality; such cannibalization problem gives an incentive to a SE to degrade organic search quality; furthermore, when consumers exhibit loyalty, SEs pro-competitively stimulate downward pressure on equilibrium quality.

Simple model

There are two search engines, *g* and *m*, which offer their search results to visitors at zero cost. Given a query, each search engine displays two links: one organic link (O-link) and one sponsored link or an advertisement (A-link). An O-link is denoted as O_i and a sponsored link as A_i at site i.[4] A unit mass of homogeneous, risk-neutral consumers search on the Internet to satisfy the needs. The cost to visit or revisit each search engine is $S > 0$. When search results are displayed to the consumer, she can further incur search cost, $s > 0$,

[4] In his study, Taylor uses a single link as a proxy for each of the lists of O- and A-links. It allows representing an explicit and tractable consumers' optimal click order, where consumers are able to substitute between the two types of link.

for each clicked link. If a consumer finds what she looks for, and her need is satisfied, she gets an expected surplus normalized to 1. Expected match probabilities are statistically independent across links and consumers. Since consumers exhibit unit demand, they do not continue searching once being satisfied.

Using the ranking algorithm that provides the distribution over match probabilities for organic link O_i, a SE chooses quality p_i that refers to the expected match probability for the O-link. Such quality $p_i \in [0, p^{\max}]$ with $p^{\max} \in [s,1]$, where $p^{\max}$ denotes the maximum technologically feasible quality. The provision of quality p_i is assumed to be costless.

Taylor also assumes that the same link is displayed as an A-link at both search engines. Each time an A-link is clicked, search engine *i* gets an amount of *b*, where *b* is the second highest bid of a player in the second price auction. In order to allocate an ad among sponsored links, advertiser *j* makes a bid according to the second price auction and expects to get $v_j \psi_j$, where v_j is what an advertiser gets per match, and ψ_j is the proportion of the visitors, who execute the transaction. Since consumers don't click more than once on the same link because it is costly, an advertiser *j* has a weakly dominant strategy to multi-home and to bid $b_j = v_j \psi_j$ per click at both search engines. If an advertiser submits the maximum bid in both ad-auctions, she will be a winner twice. According to such bidding, the expected match probability (quality) for the paid link is $q = E(\max \psi_j)$, which is taken by the search engine as given.

The variety of links, displayed for consumers, is the following: there are one A-link and two different O-links. Consumers are not aware of the match probabilities while they don't click, but they are aware of the average match probabilities p_g, p_m, and q due to the regular search at both search engines. If the quality of paid links is less than the search cost per click, $q < s$, then no consumer visits an A-link, and a search engine gets zero profit that makes it to be indifferent towards the choice of O-link quality, *p*. If $s \leq q < S + s$, consumers click on paid links iff organic link quality is able to compensate costs, *S*, for visiting a SE, and, finally, if $S + s \leq q < 1$, consumers always want to click on an A-link.

Therefore, the game is constructed as follows: SEs move first, simultaneously selecting quality p_i. Consumers define their strategy concerning a click order, observing p_g, p_m, *q*, *S*, and *s*. The game ends when all the consumers are satisfied, or they have visited all the proposed sites without finding what they need, or they just have stopped searching without meeting the needs.

2.2.1 Equilibrium behaviour. Low quality organic results: minimal quality and revenue cannibalisation

Having a threat of cannibalization, a search engine can degrade its organic results quality by fixing it below the maximum technologically feasible level. In equilibrium, rational consumers click on links in the order, which allows maximizing their expected utility. Thus, the optimal click order will involve visiting the highest quality websites first. Taylor gives an explicit description of such an optimal click order.

Hence,

$$e \equiv \begin{cases} g \text{ if } p_g > p_m \text{ and } p_g \geq s, \\ m \text{ if } p_g < p_m \text{ and } p_m \geq s, \\ g \text{ w.p.}1/2, m \text{ w.p.}1/2 \text{ if } p_g = p_m \text{ or } \max\{p_g, p_m\} < s, \end{cases}$$

where e is the search engine, which a consumer prefers to visit first, p_g and p_m are the qualities of organic links of two search engines g and m, and s are consumer search costs.

The system of equations summarizes *three cases* when a consumer's optimal click order is different. In the first case, if an organic link quality of SE g is higher than one of SE m, and consumer's benefits from visiting organic link at g cover clicking costs s, a consumer visits SE g first. In the second case, if organic link quality of SE m is higher than one of g, a consumer visits SE m first. In the third case, if organic link qualities of both SEs are equal, a consumer is indifferent which SE to visit first. Hence, she visits g and m with equal probabilities.

Given q as sponsored link quality and S as costs, paid each time for visiting (or re-visiting) a search engine, a consumer's search *strategy* is the following:

Suppose that $q \geq S + s$. Any consumer's best response strategy maps the link qualities $\{p_g, p_m, q\}$ into a link order $\{a_1, a_2, a_3\}$ in the following manner. Begin by clicking a_1 thus:

$$a_1 = \begin{cases} O_e \text{ if } p_e > q, \\ A_e \text{ if } p_e < q, \\ A_e \text{ w.p. } \lambda, O_e \text{ w.p.} 1-\lambda \text{ if } p_e = q. \end{cases}$$

a_1 *assumes three cases:*

1. A consumer clicks on an organic link first if organic link quality is higher than paid link quality.
2. A consumer clicks on a paid link first if organic link quality is lower than paid link quality.
3. A consumer clicks on a paid link first with probability λ and an organic link first with probability (1 - λ) if organic link quality and sponsored link quality are equal.

If a consumer's need was met by a_1 then stop clicking (i.e $a_2 = a_3 = \text{ø}$), otherwise click a_2 :

$$a_2 = \begin{cases} A_e \text{ if } a_1 = O_e \text{ and } p_{-e} < \sigma q, \\ O_{-e} \text{ if } a_1 = O_e \text{ and } p_{-e} > \sigma q, \\ A_e \text{ w.p.} \lambda, O_{-e} \text{ w.p.} 1 - \lambda \text{ if } a_1 = O_e \text{ and } p_{-e} = \sigma q, \\ O_e \text{ if } a_1 = A_e \text{ and } p_e \geq s, \\ \text{no clicks if } p_e < s, \end{cases}$$

where $\sigma = (S + s)/s$ is the relative cost of switching.

a_2 assumes five cases:

1. Given that to switch a SE is more expensive than to continue search at the current one, a consumer will click on a paid link of the current SE if she has clicked on an organic link of the current SE earlier and hasn't met her need.
2. Given that the relative switching costs can be covered by the benefits from the visiting an organic link of the alternative SE, a consumer switches the SE and clicks on an organic link of the alternative SE if she has clicked on an organic link of the current SE earlier and hasn't met her need.
3. Given that a consumer is indifferent between switching the SE and staying at the current one, she will click on a paid link of the current SE with probability λ and on an organic link of the alternative SE with probability (1 - λ) if she has clicked on an organic link of the current SE earlier and hasn't met her need.
4. Given that benefits from visiting an organic link can cover clicking costs, a consumer visits an organic link of the current SE if she has clicked on a paid link of the current SE earlier and hasn't met her need.
5. If consumer's further expected benefits are less than further clicking costs, a consumer stops her search even not being satisfied.

If the consumer's need was met by a_1 or a_2 then stop clicking, otherwise click a_3:

$$a_3 = \begin{cases} O_{-e} \text{ if } a_2 \neq O_e, \text{and } p_{-e} \geq S + s, \\ A_{-e} \text{ if } a_2 = O_{-e}, \\ \text{no clicks if } p_{-e} < S + s. \end{cases}$$

a_3 assumes three cases:

1. Given that the benefits from visiting an organic link of the alternative SE can cover switching costs and clicking costs, a consumer switches the SE and clicks on an organic link of the alternative SE if she has visited the current SE and hasn't met her need.
2. A consumer clicks on a paid link of the alternative SE if she has clicked on an organic link of the alternative SE earlier and hasn't met her need.
3. A consumer stops her search if the benefits from the clicking on organic link quality of the alternative SE are less than further clicking costs and switching costs.

Since a consumer clicks on the highest quality link first, a SE gets an incentive to increase its quality results to attract more visitors. In the case when a SE has its paid link of the highest quality and an organic link of the sufficiently low quality, there is no cannibalization problem, since a consumer clicks on a sponsored link first. Therefore, it becomes clear that, in equilibrium, for an organic link quality to be lower than its sponsored counterpart, it must be established a technological constraint.

Lemma 1 'when organic link quality is constrained to be below sponsored link quality (when $p^{\max} < q)$, equilibrium quality must be set at the maximum feasible level: $p_g = p_m = p^{\max}$',

where p_g, p_m are average match probabilities (qualities) of organic links of SEs g and m, and q is the quality of a sponsored link.

Proof

For simplicity, two SEs are denoted as *i* and *–i*. If $p_i \leq p_{-i} < p^{\max} \leq q$, then *i*'s A-link is clicked with the probability that is less than 1, and SE *i* has a profitable deviation in increasing $p_i \in (p_{-i}, p^{\max})$. If $p_i < p_{-i} = p^{\max}$, then, according to the above described search strategy, a consumer will prefer to visit SE *–i*, and all A-link clicks will be done at SE *–i*, and SE *i* will make zero profits.

However, if SE *i* increases p_i to $p_i = p_{-i} = p^{\max}$, it can make positive profits that are $[\lambda + (1-\lambda)(1-p^{\max})]b/2 > 0$, when $p^{\max} = q$, where $(1-\lambda)(1-p^{\max})$ is the joint probability of two independent events: a consumer clicks on an organic link with probability $(1-\lambda)$, and her need is not met with probability $(1-p^{\max})$. Profits *b/2* > 0 when $p^{\max} < q$, where *b/2* implies that one half of consumers will click on an A-link of SE *i*, and other half will click on an A-link of SE *–i*, since $p_i = p_{-i} = p^{\max}$.

Lemma 2 'when the maximum feasible organic link quality exceeds the sponsored link quality (when $p^{\max} \geq q$), any equilibrium must have $p_g = p_m \geq q$ (search engines set symmetric organic link quality not less than sponsored link quality)'.

Proof

Suppose, in equilibrium a consumer has the strategy, which is described above. If $p_i > p_{-i}$ and $p_i \leq q$, then SE *–i* makes zero profits, because a consumer never visits it, but this SE can make a profitable deviation by setting $p'_{-i} \geq \max\{s, p_i\}$.

When $p_i > q$ and $p_{-i} < \sigma q$, SE *i* gets profits of (1 - p_i)*b* , that is decreasing in p_i, where (1 - p_i) is the probability that a consumer will not satisfy her need by clicking on an organic link of SE *i*. If $p_{-i} = \sigma q$, then SE *i*'s profits are $\lambda(1-p_i)b$ (a consumer clicks on an A-link of SE *i* with probability λ and with probability (1 - λ) she clicks on O-links of *–i*) that again decrease in p_i.

If $p_i > p_{-i} > \sigma q$, then a consumer has the click order $\{O_i, O_{-i}, A_{-i}\}$ and SE *i*'s profits are zero, since a consumer doesn't not come back at SE *i* because of switching costs, given that she hasn't satisfied her need at the organic side of SE *–i*. The profitable deviation for SE *i*, in this case, is to decrease organic link quality $p'_i \in (\sigma q, p_{-i})$. Therefore, when

$p^{\max} > q$, $\min\{p_g, p_m\} < q$ implies $p_g = p_m = p < q$. In order to be more competitive SE $i \in \{g, m\}$ imposes $p_i' \in (p, q)$ to attract more consumers.

If organic link quality exceeds sponsored link quality, more consumers start to substitute search on a sponsored side by an organic side. Thus, the higher organic link quality will be, the larger cannibalisation problem will take place. Hence, a search engine will push equilibrium quality to the minimum admissible level, but still it will try to keep this admissible level higher than at the competitive SE.

Equilibrium 1 (Low quality cannibalisation equilibrium) The lowest quality, which can ever be sustained in equilibrium, has organic link quality set equal to sponsored link quality ($p_g = p_m = q$). Such an equilibrium can be supported whenever advertisements are sufficiently useful, when $q \geq 1/2$.

Proof

Given the described above consumer strategy, low quality cannibalisation equilibrium can be proved in the following way:
when $p_g = p_m = q$, SE *i*'s expected profits are $\pi_i = (\lambda + (1-\lambda)(1-p_i))b/2$.
If SE *i* deviates to $p_i' < p_{-i} = q,$ the consumer's click order is $\{A_{-i}, O_{-i}, O_i\}$, and SE *i* gets zero profits, since its A-link is never clicked. Vice versa, if SE *i* sets $p_i' > p_{-i} = q$, the consumer's click order becomes $\{O_i, A_i, O_{-i}\},$ and SE *i* gets deviation profits of $\pi_i' = (1-p_i')b,$ while SE *–i* gets zero profits.
For the deviation (from $p_i = q$ to $p_i' > p_{-i} = q$) to be non-profitable, π_i must be greater than π_i' that confirms when $q \geq 1/(1+\lambda)$ (found by substitution *q* on the places of p_i and p_i'):

$$\pi_i \geq \pi_i'$$
$$(\lambda + (1-\lambda)(1-q))b/2 - (1-q)b \geq 0$$
$$q \geq 1 - \lambda/(1+\lambda)$$
$$q \geq 1/(1+\lambda)$$

Since a consumer is indifferent in her click order, when $p_g = p_m = q$, any λ implies best response. Thus, suggested strategies form an equilibrium with $q \geq 1/2$.

Decreasing the organic link quality, a search engine faces the trade-off. On the one hand, consumers start to click more intensively on sponsored links, but on the other one, they are not satisfied with the quality of organic search quality and can switch on the other search engine. Nevertheless, *low quality cannibalisation equilibrium* claims sponsored link quality to be a significant factor in affecting SE's profits. The search engine's investments in increasing of the quality of sponsored links, such as refinement of the quality scoring algorithms, direct to make tacit collusion around low organic link quality/high profit equilibria.

2.2.2 Equilibrium behaviour. Higher qualities: switching, sticking, and consumer lock-in

Along with the *low quality equilibrium*, Taylor considers also the *high quality equilibrium*. To understand the concept of high equilibrium, it is assumed that *consumer's clicking costs are close to zero* that allows visiting each additional link without incurring any losses. Thus, a consumer will prefer not to switch SEs (not to pay switching costs), but exhaust all the costless clicks at the current search engine. Since an A-link will be clicked only at the current SE, search engines will compete for capturing all such clicks by increasing search results quality. This situation much looks like Bertrand price competition, when the player (the SE in our case), which proposes a lower price (higher quality in our case), captures the whole market.

Remark 1 'when within-site link *clicking costs, s, approach zero*, there exists a *high quality equilibrium*, in which search engines set the maximum technologically feasible quality: $p_g = p_m = p^{\max}$'.

In the case when not only *clicking costs are low*, but also *switching costs approach zero*, consumers can gain a lot by switching from one SE to another. Their expected utility of such 'switching' when $q < p_g \le p_m$, implying that a consumer visits p_m first and then switches to p_g, can be expressed as

$$U_{switch} = \overbrace{p_m(1-S-s)}^{\textit{Satisfied by } O_m} + \overbrace{(1-p_m)p_g(1-2S-2s)}^{\textit{Satisfied by } O_g} + \overbrace{(1-p_m)(1-p_g)q(1-2S-3s)}^{\textit{Satisfied by } A_g} + \\ + \underbrace{(1-p_m)(1-p_g)(1-q)(-2S-3s)}_{\textit{Unsatisfied}}, \tag{2.2.2.1}$$

where (1 – $f(S,s)$) are consumer's net benefits.

On the contrary, when switching costs are not compensated by the expected benefits from switching to SE *g*, a consumer will prefer to 'stick' and click on an O-link and an A-link at SE *m*. In this situation, consumer's expected utility is the following:

$$U_{stick} = \overbrace{p_m(1-S-s)}^{\text{Satisfied by } O_m} + \overbrace{(1-p_m)q(1-S-2s)}^{\text{Satisfied by } A_m} + \overbrace{(1-p_m)(1-q)p_g(1-2S-3s)}^{\text{Satisfied by } O_g} + \\ + \underbrace{(1-p_m)(1-q)(1-p_g)(-2S-3s)}_{\text{Unsatisfied}}. \qquad (2.2.2.2)$$

A consumer is indifferent between 'switching' and 'sticking' when her expected utilities are equal. Hence, setting $U_{\text{switch}} = U_{\text{stick}},$ it is found that a consumer is indifferent when

$$-qS - qs + p_g s = 0$$
$$\frac{p_g}{S+s} = \frac{q}{s}.$$

Clicking the next link at the current search engine, a consumer incurs costs, *s*. Instead, if she decides to switch on the alternative SE, she will pay *S*+*s* for the click on the next link. Thus, the relative costs of switching versus sticking are $\sigma \equiv (S+s)/s$. In order to express consumer's indifference between switching and sticking, one assumes the same benefit-cost ratio, whenever a consumer stays at the same search engine or she switches to the alternative one. By the rearrangement, the threshold value of p_g is

$$p_g = \frac{S+s}{s} q \equiv \sigma q. \qquad (2.2.2.3)$$

When a consumer is not indifferent, *two cases* must be considered. *When switching costs are high* $(p_g < \sigma q)$, *a consumer prefers to stick* since switching is rather expensive. On the contrary, *when switching costs are low* $(p_g > \sigma q)$, *a consumer prefers to switch* after visiting first an O-link.

Depending on the type of switching costs (high or low), SEs' strategies are different. In the situation with *high switching costs*, in equilibrium, a *SE will establish the maximum technological feasible quality* in order to push a consumer to choose this SE and lock-in. In this case, a SE will get all the clicks on an A-link. In the situation when *switching costs are* *low*, SEs will loose their incentives to 'fight' for a consumer and *will lower organic link quality* in order to get clicks on an A-link from those visitors, who switch from the rival SE. It will last till the moment when attracting an additional switcher to click on an A-link is no longer possible.

Summing up these two cases, it can be proved that there are two types of equilibrium: *Maximal quality equilibrium* and *Reduced quality switching equilibrium*.

Equilibrium 2a (Maximal quality equilibrium) 'There exists an equilibrium in which search engines set the maximum feasible quality $(p_g = p_m = p^{\max})$ if and only if relative switching costs, σ, or sponsored link quality, *q*, are sufficiently high – specifically when $\sigma q \geq p^{\max}$'.

Equilibrium 2b (Reduced quality switching equilibrium) 'If relative switching costs, σ, or sponsored link quality, *q*, are sufficiently low (specially if $\sigma q < p^{\max}$), there exists an equilibrium in which both search engines set quality σq. This is then the highest sustainable equilibrium quality'.

Existence of the *Maximal quality equilibrium* can be proved in the following way:

1. Sufficiency of $p^{\max} \leq \sigma q$:

   Let $p^{\max} \leq \sigma q$. Assume that a consumer's strategy corresponds to the one, described above. Organic link quality can not be higher than its maximal technologically feasible level ($p_i > p^{\max}$ is not possible). If a SE deviates to $p_i' < p^{\max} (< \min\{p_{-i}, \sigma q\})$, a consumer never clicks A_i, since organic link quality of SE *i* is less than one of SE –*i*, and in the conditions of high switching costs, a consumer never switches to lower quality SE. Thus, the profits of SE *i* are zero.

2. Necessity of $p^{\max} \leq \sigma q$:

   Let $p^{\max} > \sigma q$. If $p_g = p_m = p^{\max}$, then a consumer will switch between two SEs since benefits from high organic link quality will cover the switching costs, and, as a result, SE *i* will end up with the profits $\pi_i = 1/2(1 - p^{\max})^2 b$, where the term $(1 - p^{\max})^2$ implies probability that a consumer's need will not be met when she visits an organic side of both SEs. That's why a consumer will continue searching by clicking on an A-link. *1/2b*, as previously has been explained, defines what SE *i* gets, assuming that one half of consumers will click on its A-link, and another half of consumers will click on an A-link at SE -*i*.

SE i has a profitable deviation in $p_i' \in (\sigma q, p_{-i})$, that implies SE i's profits become $(1-p^{\max})(1-p_i')b$. It means that if SE i decreases the quality of its organic link, then, switching to this SE, a consumer will more likely satisfy her need by clicking on an A-link, given that she hasn't met her need by clicking O-links of both SEs. Obviously, when $p^{\max} \le \sigma q$, SEs don't decrease their organic link qualities and fix them on the maximal feasible level.

Existence of the *Reduced quality switching equilibrium* can be *proved* in the following way:

when $p_g = p_m = \sigma q > q$, the expected profit of SE i is

$\pi_i = 1/2[\lambda(1-p_i) + (1-\lambda)(1-p_{-i})(1-p_i)]b$

that means that half of the consumers first click on an O-link of SE *i,* and if they don't find what they look for, with the probability λ they click on an A-link of SE *i*. With the probability (1 - λ) consumers, who are not satisfied after clicking on an organic link of SE *i*, will click on an O-link of SE -*i*, given that the expected benefit from clicking on such O_{-i} is enough to cover switching costs. If consumers don't satisfy the needs clicking on an O-link of SE –*i,* they will click on its A-link (that will be counted to the profits of SE –*i*, whereas other half of consumers will finally click on an A-link of SE *i,* not having met their needs on SE –*i,* and on SE *i's* organic side*)*.

SE i's deviation to $p_i' < p_{-i} = \sigma q$ gives it zero profits since consumer never click on an A-link of SE *i,* if $p_i < \min\{\sigma q, p_{-i}\}$. Instead, if SE i decides to deviate $p_i' > p_{-i} = \sigma q$, its profits become $\pi_i' = \lambda(1-p_i')b$ (consumers will prefer to visit SE i first). Since the profit is, in any case, decreasing in p_i', it would be logical to consider a limiting case, when $p_i' = p_{-i} = \sigma q$. The deviation to π_i' is not profitable while $\pi_i \ge \pi_i'$. Substituting σq in the places of both p_i' and p_i, $\pi_i - \pi_i' \ge 0$ becomes:

$1/2[\lambda(1-\sigma q) + (1-\lambda)(1-\sigma q)(1-\sigma q)]b - \lambda(1-\sigma q)b \ge 0$,

and by rearranging yields $\lambda \le (1-\sigma q)/(2-\sigma q)$. This condition is consistent with the equilibrium, since consumers are indifferent over all λ, when $p_g = p_m = \sigma q$. Moreover, according to *low quality cannibalisation equilibrium* $p_g = p_m$, if $p_g = p_m = p > \sigma q$ with the SE's profits $1/2(1-p)^2 b$, there is a profitable deviation to decrease organic link quality to $p' \in (\sigma q, p)$ implying profits $(1-p)(1-p')b$. Hence, it is the situation when $p > \sigma q$ cannot be an equilibrium.

Depending on the size of switching costs, equilibriums are different. A consumer can prefer to 'lock in' or to switch between SEs. The impact from the increase of switching costs can be examined by taking consumers' utility (2.2.2.1) and substituting equilibrium quality level, given in (2.2.2.3), into (2.2.2.1), and to differentiate consumer's utility with respect to the visiting cost, *S*.

Thus, U_{switch} becomes

$$U_{switch} = \frac{S+s}{s}q(1-S-s) + (1-\frac{S+s}{s}q)(\frac{S+s}{s}q)(1-2S-2s) +$$

$$+(1-\frac{S+s}{s}q)(1-\frac{S+s}{s}q)q(1-2S-3s) + (1-\frac{S+s}{s}q)^2(1-q)(-2S-3s).$$

$$\partial U / \partial S = \frac{2(1-q)(q-s)((1-q)s-qS)}{s^2} \geq 0,$$

given that $\sigma q \leq 1$, it implies $((1\text{-}\ q)s - qS) \geq 0$,

since by rearranging

$$\sigma q \leq 1$$

$$\frac{s+S}{s}q \leq 1$$

$$q(s+S) \leq s$$

$$s - qs - qS \geq 0,$$

is equivalent to $((1\text{-}\ q)s - qS) \geq 0$. Moreover $(q - s) > 0$ (otherwise consumers never click on an A-link). Since the derivative is positive, and higher switching costs imply higher utility, one can claim that:

Remark 2 *'Increases in switching costs can induce higher equilibrium quality and make consumers better-off'.*

Therefore, Taylor (2013) considers three types of equilibrium: *low quality cannibalisation equilibrium, maximal quality equilibrium and reduced quality switching equilibrium*. These three equilibriums are explained by the distinct forces in the search market (revenue cannibalisation, competition for a market share, and intra-query switching). The support of each equilibrium depends on the size of switching costs and the quality of sponsored search results.

2.2.3 Consumers with search engine "loyalty"

The results of the previous section show that there exists not only the equilibrium with the degraded quality (*low quality cannibalisation equilibrium*), but also the stark equilibrium (*maximal quality equilibrium*), in which SEs establish their organic link quality on the maximum feasible level. Such stark equilibrium exists only with the assumption of the discontinuous demand, where an arbitrarily small quality advantage is sufficient to capture the entire market (similarly to Bertrand competition).

Certainly, in reality, consumers don't react so sharply to the changes of search quality since they tend to exhibit some level of loyalty. But, if it is allowed that demand reacts smoothly on the change of the quality, a new question arises: 'What quality of O-links can be sustained if there is some degree of continuity to the demand faced by each search engine'?

To get an answer, Taylor imposes a standard Hotelling linear city model where consumers have heterogeneous visit costs, distributed along the segment of the real line. He assumes that consumers are uniformly distributed along a line of unit length with the location of *g* at the point 0, and *m* at point 1. A consumer with the location at point *x* (where *x* are individual preferences of a consumer about a SE) pays cost $S_g(x) = Sx$ to visit SE *g* and $S_m(x) = S(1-x)$ to visit SE *m*, where *S* can be denoted as transport cost. It is assumed that $1 > q > S + s$ that means that each consumer would have a willingness to click on an A-link. In addition, it is assumed that clicking costs, *s*, are small.

Depending on the structure of visit costs and personal preferences, a consumer decides, which SE to visit first. Particularly, for some x^* consumer's best response is to visit *g* first if $x \le x^*$ (otherwise click *m* first) and click firstly on the link of higher quality (an O-link or an A-link). A consumer will stop clicking when her need is satisfied or when O-link quality of the alternative SE is too small. Obviously, setting $p_i < S$ can take consumers far away from SE *i*. Hence, SE *i* should set the floor for its organic link quality in order not to lose consumers. It is possible to show that sponsored link quality can play a role of such floor for equilibrium organic link quality.

Lemma 3 'Offering an organic link whose quality is below that of the sponsored link (setting $p_i < q$) is a dominated strategy for the search engines.'

Proof

Let $p_g < q$. Given that clicking costs are small, the mass of consumers x^* (where $x^* > 0$) will click on A_g (A-links of SE g) only if they visit g first. If p_g is the best response strategy to attract consumers, then SE g will encourage nearby consumers (according to Hotelling model) to visit it first by establishing p_g close to p_m.

When $0 < x^* < 1$, given $p_g < q$, rational nearby consumers will prefer to click on g's A-link first. The profits of g, in this case, will be x^*b. The click orders of the consumers are the following: $\{A_g, O_g, O_m\}$ or $\{A_g, O_g, \varnothing\}$ with the correspondent utility functions:

$$U(A_g, O_g, O_m) = q(1-Sx) + (1-q)p_g(1-Sx) + (1-q)(1-p_g)p_m(1-S) -$$
$$-(1-q)(1-p_g)(1-p_m)S,$$

and $U(A_g, O_g, \varnothing) = q(1-Sx) + (1-q)p_g(1-Sx) - (1-q)(1-p_g)Sx$,

where S are transport costs that consumers incur when they switch the SE. When consumers visit SE g, they pay Sx; when x^* mass of consumers switch to SE m (that means that now all the consumers (x^* and $(1- x^*)$) are at m, and the total mass of all consumers become close to 1), all of them pay
$S(1 - x^* + x^*) = S$.

Other rational consumers, who prefer to visit m first, rather than g, have the following click orders: $\{A_m, O_m, O_g\}, \{A_m, O_m, \varnothing\}, \{O_m, A_m, O_g\}$, and $\{O_m, A_m, \varnothing\}$ with the corresponded utility functions:

$$U(A_m, O_m, O_g) = q(1-S(1-x)) + (1-q)p_m(1-S(1-x)) + (1-q)(1-p_m)p_g(1-S) -$$
$$-(1-q)(1-p_m)(1-p_g)S,$$

$$U(A_m, O_m, \varnothing) = q(1-S(1-x)) + (1-q)p_m(1-S(1-x)) - (1-q)(1-p_m)S(1-x),$$

$$U(O_m, A_m, O_g) = p_m(1-S(1-x)) + (1-p_m)q(1-S(1-x)) + (1-p_m)(1-q)p_g(1-S) -$$
$$-(1-q)(1-p_m)(1-p_g)S,$$

$$U(O_m, A_m, \varnothing) = p_m(1-S(1-x)) + (1-p_m)q(1-S(1-x)) - (1-p_m)(1-q)S(1-x).$$

Given that $S_g(x)$ is continuously increasing in x, consumers' mass, who click on A_g first, must be $[0, x^*]$. In addition, there exists a marginal consumer at x^*, who is indifferent between search at SE g or SE m first, or between clicking orders, in general; contrary to other consumers at $x+\epsilon$ and $x - \epsilon$, where ϵ is small. Thus,

$$\max\{U(A_g, O_g, O_m), U(A_g, O_g, \emptyset\}=$$

$$= \max\{U(A_m, O_m, O_g), U(A_m, O_m, \emptyset), U(O_m, A_m, O_g), U(O_m, A_m, \emptyset)\} \text{ at } x = x^*.$$

Considering a small increase in p_g to $p'_g \in (p_g, q)$, it is easy to verify that

$$\min\left\{\frac{\partial U(A_g,O_g,O_m)}{\partial p_g}, \frac{\partial U(A_g,O_g,\emptyset)}{\partial p_g}\right\} \geq$$

$$\max\left\{\frac{\partial U(A_m,O_m,O_g)}{\partial p_g}, \frac{\partial U(A_m,O_m,\emptyset)}{\partial p_g}, \frac{\partial U(O_m,A_m,O_g,)}{\partial p_g}, \frac{\partial U(O_m,A_m,\emptyset)}{\partial p_g}\right\}$$

where

$$\frac{\partial U(A_g,O_g,O_m)}{\partial p_g} = (q\text{-}1)[S(x\text{-}1)+p_m\text{-}1]$$

$$\frac{\partial U(A_g,O_g,\emptyset)}{\partial p_g} = 1\text{-}q$$

$$\frac{\partial U(A_m,O_m,O_g)}{\partial p_g} = (q\text{-}1)(p_m\text{-}1)$$

$$\frac{\partial U(A_m,O_m,\emptyset)}{\partial p_g} = 0$$

$$\frac{\partial U(O_m,A_m,O_g)}{\partial p_g} = (q\text{-}1)(p_m\text{-}1)$$

$$\frac{\partial U(O_m,A_m,\emptyset)}{\partial p_g} = 0$$

Hence, it is easy to see that even a small increase in organic link quality of SE g makes a consumer better off when she visits SE g first. Therefore, if SE g increases p_g, setting $p'_g \in (p_g, q)$, then the marginal consumer will strictly prefer the click order that implies visiting A_g first. Hence, the mass of consumers and the profits of SE g will also increase. Given the continuity of p_g, such effect from p_g's increasing will work for all $p_g < q$.

Given that organic link quality is initially low, there is no threat of cannibalization problem because consumers prefer to visit a sponsored link first. Moreover, it is possible to identify the location of the marginal consumer (x^*), who is indifferent between visiting SE g or SE m first. Thus, location x^* is the solution to any pairwise equation of utilities from click orders with two clicks at the firstly visited search engine, as for example to $U(O_g, A_g, O_m,$ $\varnothing) = U(O_m, A_m, O_g, \varnothing)$:

$$p_g(1-Sx^*)+(1-p_g)q(1-Sx^*)+(1-p_g)(1-q)p_m(1-S)+(1-p_g)(1-q)(1-p_m)(-S)=$$
$$=p_m(1-S(1-x^*))+(1-p_m)q(1-S(1-x^*))+(1-p_m)(1-q)p_g(1-S)+$$
$$+(1-p_g)(1-q)(1-p_m)(-S),$$

that simplifies to

$$-p_gSx^*-qSx^*+p_gSx^*q=p_mSx^*-Sq+qSx^*-p_mqSx^*-p_gS+qp_gS$$

with the further simplification to

$$\frac{x^*}{1-x^*}=\frac{p_g+(1-p_g)q}{p_m+(1-p_m)q}, \qquad (2.2.3.1)$$

implying that relative market shares of the two search engines are defined by the relative probabilities of offering the link that will satisfy a consumer, i.e. higher quality of a SE corresponds to the higher relative market shares. Therefore, given that SE g's market share, x^*, is increasing in p_g, it becomes clear that there is no equilibrium with the setting organic link quality below q that proves *Lemma 3*.

Furthermore, Taylor (2013) considers equilibrium where organic link quality is set at least as high as its sponsored counterpart when online search engines jointly fix their organic link quality on the minimum admissible level.

Equilibrium 3 'There exists a $\underline{q}$ such that there is an equilibrium when organic link quality is set equal to sponsored link quality $(p_g = p_m = q)$ whenever $q \geq \underline{q}$ (when sponsored link quality is rather high).'

Since sufficiently high quality of the sponsored link facilitates the existence of the low-quality equilibrium, ***Equilibrium 3*** is noticeably analogues to ***Equilibrium 1***.

On the contrary, ***Equilibrium 4*** corresponds to the high quality equilibrium:

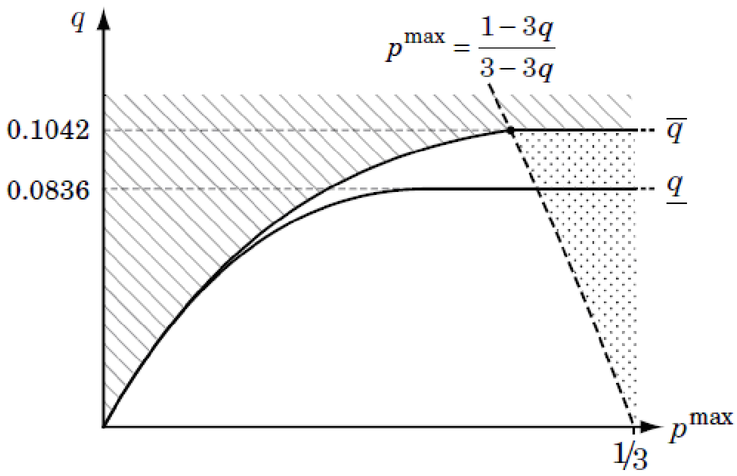

Figure 2.2.3.1.Values of $\bar{q}$ and $\underline{q}$ given various technological ceilings, $p^{\max}$, and sponsored link quality, q (Taylor, 2013)

Equilibrium 4 'There exists a $\bar{q}$ such that an equilibrium when organic link quality is strictly greater than sponsored link quality can be supported if and only if $q \leq \bar{q}$ (when sponsored link quality is rather low).'
The unique equilibrium corresponds to the quality equal to

$$\frac{1-3q}{3-3q} \tag{2.2.3.2}$$

when it is technologically feasible, and equal to the maximum feasible quality, $p^{\max}$, otherwise.

In Figure 2.2.3.1 it is summarized the form of $\underline{q}$ and $\bar{q}$. $\underline{q}$ corresponds to the lower solid line; the locus of points above this line indicates no profitable deviation from Equilibrium 3. Instead, the locus of points below the solid line of $\bar{q}$ shows no profitable deviation from Equilibrium 4.

When the search engine increases its organic link quality, consumers start to substitute their clicks on the sponsored links by the organic links that will hurt the search engine's welfare. However, if the maximum feasible quality of an organic link is rather low, the search engine can increase its organic link quality without facing problems in terms of cannibalization. Since organic link quality is much lower than sponsored link quality, consumers will not be able to satisfy their needs on the organic side and will continue to click on sponsored links. However, such improvement of the organic link quality will proceed till the moment when further rise of quality will lead to the revenue cannibalization. Thus, regardless of the rival's strategy, at the definite time it will be rational for the search engine to stop the improvement of its ranking algorithm[5].

The Figure 2.2.3.1 shows that, when maximum feasible organic link quality is not decreasing over time, the industry must be concentrated either in the dotted region, or in the hatched region that can not support the high quality equilibrium.

Apart from this, high quality Equilibrium 4 induces the organic link quality to be (weakly) decreasing in sponsored link quality. It can be checked by substituting the equilibrium organic link quality (2.2.3.2) into consumers' utility (2.2.2.2) and differentiating with the respect to sponsored link quality:

$$\frac{\partial U}{\partial q} = -\frac{4}{9(1-q)^2} < 0.$$

Thus, equilibrium consumers' utility is decreasing in sponsored link quality when maximum technologically feasible organic quality is set. Despite consumers directly benefit from the increase in sponsored link quality, their gain is neutralized by the absence of the SE's incentive to provide higher organic link quality. Therefore, consumers end up with the substantial payment for the sponsored link quality improvement.

Remark 3 *Improvements in sponsored link quality can reduce equilibrium quality and make consumers worse-off.*

When the quality of the sponsored link increases, consumers satisfy their needs on the sponsored side and become less sensitive to the organic link quality. Clearly, in this situation, given that sponsored link quality is high, the SE's market share depends less on the organic link quality. That's why it is always optimal for the SE to sustain the lower organic search quality in equilibrium.

2.2.4 Conclusions and critical remarks

Summing up, Taylor (2013) examines equilibrium behaviour of the Internet search engine in a model of the Internet search market and considers the case when competition for

[5] The SE's profit can be obtained from the following equation: $\pi = (1-p)x^*b$. Since, when $p \to 1, \pi \to 0$, the SE will always degrade its organic link quality.

consumers' clicks pushes SEs to provide higher quality search results. Taylor finds that, when consumers are rational and choose which SE to visit first according to the link relevance, a search engine will always try to increase organic results quality to encourage consumers to visit it first. In this situation, if organic link quality becomes too high, a SE can face the cannibalisation problem when consumers prefer to click on organic links first and consequently satisfy their needs, rather than click on sponsored links – the revenue source for the SE. Hence, if SE starts reducing organic link quality, new equilibrium with the degraded result quality will take place. When consumers exhibit loyalty, low quality equilibrium becomes stark. Nevertheless, Taylor shows that setting organic link quality below sponsored link quality is dominated strategy, because even a small increase in the organic quality will lead to an increase in the relative market share.

Improvement in the sponsored search results relevance helps consumers to satisfy their needs by searching on the sponsored side, but also reduces a SE's incentive to provide high quality organic search results. As a result, consumers are worse-off overall. Considering the size of the switching costs consumers incur, Taylor finds that such costs can be pro-competitive. Their existence can force consumers to 'stick' and spend more time clicking on sponsored links at the first visited search engine, rather than to switch to another one. Certainly, it will create a strong incentive for the SE to compete ex ante for consumers' clicks.

Nevertheless, Taylor gives an explicit description of the consumer's and the Internet search engine' strategies, which depend on the quality of organic and sponsored links, size of the switching and clicking costs; and gives the answer in which situation search engines can have an incentive to degrade their organic search results, Taylor doesn't describe in what way, in reality, a search engine can lower the quality of the organic link (with the usage of which tools). The Taylor's model doesn't reflect the process of the quality results degrading, as well; unless it shows how the profits of SE's and consumer welfare can be changed after search results quality "updates".

Under SE's setting of organic link quality Taylor understands how the real 'physical' quality of the website corresponds to its rank in the organic search results, assigned by the SE through the ranking algorithm. Taylor assumes that organic link quality and sponsored link quality are simply expected match probabilities that consumers will be satisfied by clicking on organic links or sponsored links respectively. Since the logic behind the interpretation of the concept of quality in terms of probability is quite complex, the article is quite difficult in its comprehension. Moreover, like Sen (2005), Taylor doesn't provide any explanation of the concept of a SE's ranking algorithm and ranking algorithm quality (algorithm efficiency and algorithm robustness). The SE's ranking algorithm is called 'proprietary algorithm' in the Taylor's paper.

The Taylor's study concentrates on the optimal strategies of search engines and by no means touches the advertisers' response strategies to their organic or sponsored link quality modification. The paper doesn't highlight the concept of search engine optimization and its role in online advertising when the SE tries to accommodate its organic search results quality to receive the maximum revenue, as well.

To refine the Taylor's study, in this dissertation, it has been made an attempt to build the model that shows in which way an Internet search engine can lower the quality (score) of the organic links, and how it will affect the probability of high quality websites to get high positions in the organic search results. Moreover, it has been investigated the impact of such organic search results manipulation on advertisers' strategies, SE's profits, and consumer welfare.

3 The model of online advertising

Having analysed the search-engine market, a majority of researchers have arrived at a conclusion that consumers have a specific behaviour in their online search. Thus, around 70-80% of the users prefer to click on organic links first, rather than to visit sponsored links.[6] Only 20-30% of the potential buyers search on the sponsored side. Other empirical studies show that 53% of the total organic clicks belong to the top one link, 15% of clicks go to the second link, and 9% goes to the third link in the top of organic search results.[7] Thus, consumers not only start their online search by clicking on organic links, but they more likely satisfy the needs after visiting two or three top websites.

Intuitively, such statistics can push online advertisers (websites) to invest more in SEO in order to get higher ranking positions in the organic search results. In their research study, Berman and Katona (2012) prove that investing in SEO is indeed an optimal *search engine marketing (SEM) strategy* for the high quality websites. Clearly, an implementation of such advertisers' strategy contradicts the policy of a search engine (SE), which has an objective to increase websites' investments into the paid placement. Thus, the strategy of a SE is to push advertisers to buy sponsored links. It can be done through the updating of a SE's ranking algorithm that will detect websites, which use black hat SEO, and punish them by decreasing such websites' organic rank. When a SE has a dominant market position, it can implement not only the punishment for black hat SEO, but, using abusive practices, lower an organic rank of the websites that invest neither in SEO nor in sponsored links.

Therefore, in this study, we present the model that describes the search engine market, where the SE manipulates its organic search results by the ranking algorithm, advertisers implement online SEM strategies to attract consumers, and consumers search online to satisfy their needs.

The model is an extension to the Sen's one (2005). The Sen's model describes the search engine marketing strategies of single-homing advertisers that improve the visibility of websites in the organic and paid search results provided to consumers. Contrary to the

[6] 24 Eye-Popping SEO Statistics. SEJ Search Engine Journal. http://www.searchenginejournal.com/24-eye-popping-seo-statistics/42665/

[7] 53% of Organic Search Clicks Go to First Link. Search Engine Watch. Kantar Media report. http://searchenginewatch.com/article/2215868/53-of-Organic-Search-Clicks-Go-to-First-Link-Study

study of Xing and Lin (2006), described in the previous section, Sen (2005) suggests the optimal strategy for advertisers who choose between investments in search engine optimization (SEO) and sponsored links. Counter intuitively, Sen's model shows that SEO is not a part of the equilibrium strategy and most advertisers prefer to invest, instead in SEO, in paid placement. It can be explained by the relatively high cost of SEO and absence of the guarantee to get high rank in the organic listing. Sen finds that, even when the costs of SEO and paid placement are the same, and search engine optimization can assure a high rank in organic search results, SEO is still not an optimal strategy for advertisers.

The main limitation of Sen's model is an absence of algorithm efficiency and algorithm robustness parameters, introduced by Xing and Lin (2006). Sen assumes that, while investing in SEO, advertisers automatically get the high rank in the organic search results. On the contrary, in reality, when a SE has high algorithm robustness, advertisers can get the high rank in the organic listing if and only if they implement white hat SEO, but not black hat SEO. In his study, Sen doesn't indicate, whether he assumes SEO to be of white hat or of black hat.

To refine the Sen's study, we extend his model assuming SE's manipulation of online search results. Moreover, in our analysis, we consider SEO is of white hat. That's why advertisers are not affected by the SE's algorithm robustness.

Our model is quite simple that allows proceeding with the analytical results, but, at the same time, it shows main characteristics of consumers' online shopping behaviour and the competition between online advertisers. The model is analyzed to determine the conditions in which online advertisers choose a specific SEM strategy in terms of the consequent profitability to get top positions in the organic search results.

The Model

The model is constructed as follows. Consumers search online for products or information by visiting websites. After preliminary investigation, consumers construct a consideration set of the advertisers who sell products or information that consumers would like to buy. Clearly, whether advertisers are in a consideration set or not depends on their organic ranking positions. Since empirical evidence shows that consumers click first on organic links, the model assumes that consumers will search on the sponsored side only if they don't satisfy the needs on the organic one.

As in the Sen's study, on the online market there are two competing advertisers (A and B), which provide homogeneous products or information. Consumers can view the advertisers' links as the search results after entering specific keywords into a search engine. Consumers' preferences for the two advertisers are determined by the incurred "transportation" or "discomfort' cost" (Hotelling, 1929).[8] According to Hotelling (1929), transportation cost is specified by the store location, cost of freight, consumers' preferences about a company's mode of doing business, different services, quality, etc. In our study, transportation cost depends on the convenience to execute a purchase transaction (security, privacy, shipping and return policies, etc.). Discomfort cost matters only in the case when consumers fully aware about advertisers' policies.

Consumers and advertisers are single-homing. There exists the monopolistic online search engine that, contrary to the Sen's model, manipulates its organic search results by updating the ranking algorithm. Such updated ranking algorithm is able to discover the smallest imperfections in advertising and, consequently, can significantly lower positions of the advertisers in the organic search results.

3.1 Advertisers' search engine marketing strategies and the strategy of the online search engine

Advertisers' search engine marketing strategies

[8] Hotelling (1929) uses the concept of the transportation cost in the following illustration: "the buyers of a commodity will be supposed uniformly distributed along a line of

FIG. 1.

length *l*, which may be Main Street in a town or a transcontinental railroad. At distances *a* and *b* respectively from the two ends of this line are the places of business of A and B (Fig. 1). Each *buyer transports his purchases home at a cost c per unit distance*. The cost of production to A and B is zero, and that unit quantity of the commodity is consumed in each unit of time in each unit of length of line. The demand is thus at the extreme of inelasticity. No customer has any preference for either seller except on the ground of price plus transportation cost. In general there will be many causes leading particular classes of buyers to prefer one seller to another, but the ensemble of such consideration is here symbolised by *transportation cost*."

Following Sen (2005), the model assumes that advertisers don't pay the SE to be displayed on the organic side, and pay only for paid placement or for SEO. Thus, they have four alternative strategies.

Strategy 1: Invest neither in SEO nor in paid placement (PP)

Implementing the first strategy, advertiser A or B prefers to invest neither in SEO nor in PP, and allocates an advertisement for free on the organic side. In order to fall into the consumers' consideration set, advertisers have to get high organic rank positions. According to our study, it happens when websites' quality and the SE's ranking algorithm quality are jointly high. Thus, the probability of an advertiser to be in the consumers' consideration set is the probability of the joint occurrence of two independent events that the website and the SE's ranking algorithm are both of high quality (Grinstead, 1952).[9]

Probability that the website of advertiser A or of advertiser B will be of high quality is denoted by α, where $0 < \alpha \leq 1$. Certainly, the probability to get a top position in organic search results is higher when the probability to be the website of high quality is higher. Probability that the SE's ranking algorithm will be of high quality is denoted by σ, where $0 < \sigma \leq 1$. Hence, the probability that an advertiser will get a top position in organic search results is denoted by $\alpha\sigma$. Clearly, if the SE has an aim to manipulate the quality of the organic search, it decreases its ranking algorithm quality that corresponds to the lower value of probability σ. Thus, the lower is a value of σ, the lower is the probability that a website will get a high rank in organic search results, $\alpha\sigma$; the stronger is the SE's manipulation. Since Sen (2005) doesn't assume SE's manipulation of the search results, the value of σ, in his study, is always equal to 1, and the probability to be highly ranked in organic search results is simply α.

[9] *Algorithm efficiency* corresponds to the ability of the SE to satisfy consumers' needs. In other words, with the high algorithm effectiveness, high quality websites get higher rank, and consumers are satisfied more likely by visiting such websites.
Algorithm robustness indicates how the ranking algorithm is vulnerable to noises. Since the usage of the black hat SEO can bias the page ranking and divert SE's revenues, a SE increases the algorithm robustness and strictly punishes websites, which implement black hat SEO, by decreasing of their pages ranking (Xing and Lin, 2004).

To increase the probability to be of high quality and, consequently, the probability to get the higher rank in organic search results, advertisers can invest in SEO (white hat SEO) that leads to the second alternative strategy.

Strategy 2: Invest only in Search Engine Optimization (SEO)

It is assumed that the cost of SEO is an amount of fixed investment, *c*. When the SE doesn't manipulate organic search results and has the ranking algorithm of the maximum technologically feasible quality, an advertiser, investing in white hat SEO, appears in the top of organic search results with the probability $\alpha\sigma$ =1, where α = 1 and σ =1. Since, in our study, we consider the case when advertisers invest only in white hat SEO and don't implement black hat SEO, websites are not affected by the SE's high algorithm robustness.[10]

When the SE decreases its ranking algorithm efficiency that reflects in lower σ, advertiser A or B has the probability to be in the top of organic search results $\alpha\sigma = 1 \times \sigma = \sigma$, where $\sigma < 1$.

Strategy 3: Invest only in Paid Placement

For simplicity, it is assumed that an amount of fixed investment in paid placement is the same as in SEO, *c*. If an advertiser decides to invest in PP, it gets a guarantee to be listed on the sponsored side. Thus, once *c* is invested, an advertiser's website is displayed in the paid-placement section. Apart from it, an advertiser still has a chance to be highly ranked on the organic side with the probability $\alpha\sigma$.

However, even if an advertiser is displayed on the sponsored side, consumers will click on the sponsored links only when both advertisers A and B are absent in the top of the organic search results. It may happen with the probability $(1 - \alpha\sigma)(1 - \alpha\sigma)$.

Strategy 4: Invest in both SEO and PP

[10] Contrary to the situation described in our study, a good example of the SE that has low robustness of the ranking algorithm, implying high vulnerability to noises, generated by black hat SEO, is Google.ru. The Google's ranking algorithm updating influences mostly websites with the content in English. Hence, websites with the content in Russian still remain unaffected by the Google's updates. Therefore, implementation of the black hat SEO remains the best strategy of the advertisers on Google.ru.

When an advertiser invests in both SEO and PP, and the SE's ranking algorithm is of high quality, an advertiser can be shortlisted in the organic search results with the probability $\alpha\sigma$ =1. Clearly, investing in both SEO and PP, advertisers incur double cost, 2*c*. When the SE doesn't manipulate organic search results, the fourth advertiser's strategy is dominated by the second strategy (investing only in SEO). The probability to be in the consumers' consideration set is 1, $\alpha\sigma$ =1, in both cases, but the implementation of the fourth strategy implies additional cost.

Despite Sen (2005) ignores the fourth strategy in the subsequent analysis because of its high cost, we decide to proceed with the inclusion of this strategy in our study since in the conditions of the SE's manipulation the probability to be in the consumers' consideration set is no more equal to 1, $\alpha\sigma$ =1, but it is much lower (since σ < 1).

The online search engine's strategy

Contrary to Sen (2005) who doesn't analyze the online search engine's strategy in his study, we assume that the online search engine has a monopolistic power, and it considers the best strategy to decrease the rank of the advertisers in the organic search results pushing them to invest in paid placement. By decreasing the probability σ through lowering the SE's ranking algorithm quality, the SE can adjust downwards the probability of the advertisers to get the high organic rank to its technologically feasible minimum. The SE gets the maximum revenue, *2c*, when both advertisers invest in PP. Whether it happens or not in the conditions of the organic search results manipulation, we can check by proceeding with the further analysis.

3.2 Demand distribution

Depending on the search engine marketing strategy, which is chosen by two advertisers, three different scenarios can be applied (Sen, 2005):

1. Both advertisers have rather low organic rank and absent from the consumers' consideration set. In this situation advertisers' profit is zero since no consumer will find two websites during online search.
2. One advertiser is in the consideration set and one is not. It can happen when one advertiser has the website of higher quality than other advertiser. In the Sen's

study, the probability to be in the consumers' consideration set is α, while the probability to be out is (1 - α). In our study, the probability to be in the consumers' consideration set is $\alpha\sigma$, while the probability to be out is (1 - $\alpha\sigma$). The probability that only one advertiser will be in the consideration set is denoted by p_i, where $i = A, B.$ This case corresponds to the situation when an advertiser captures the whole market and faces the demand $D_i^1 = 1$, where $i = A, B.$ The upper index 1 indicates the case when only one advertiser appears in the consumers' consideration set.

3. Both advertisers get the high organic rank and present in the consumers' consideration set. In the Sen's study, it happens with the probability α^2, whereas in our study - with the probability $\alpha^2\sigma^2$, where such probability is denoted by p. Such scenario corresponds to the case of a competitive duopoly market. It is assumed that consumers are uniformly distributed between 0 and 1 with the density 1. Two advertisers are located at the extremes of the consumers' 'comfort level' scale. In other words, advertiser A is located at $x = 0$, and advertiser B is located at $x = 1$. Consumers are assumed to have a total transportation cost normalized to 1. When a consumer is attracted to the 'comfort level' *x*, buying from advertiser A, she will incur a discomfort cost of *x;* and a discomfort cost of (1 – *x*), buying from advertiser B. The consumers have unit demand since one consumer buys no more than one unit of the good.

As in the Sen's study, each advertiser charges price P_i for the good, where $i = A, B.$ Thus, an indifferent between two advertisers consumer, located at *x* incurs total costs (including cost of the good and transportation cost) $P_A + x = P_B + (1 - x)$. Advertisers' respective demands are:

$$\left.\begin{aligned} D_A^2 &= x = \frac{1}{2}(P_B - P_A + 1) \\ D_B^2 &= (1 - x) = \frac{1}{2}(P_A - P_B + 1) \end{aligned}\right\}, \qquad (3.2.1)$$

where an upper index 2 indicates the case when both advertisers present in the consumers' consideration set.[11]

[11] Table. The interpretation of notations

p_i $(i = A, B)$	the probability that only one advertiser will be in the consideration set
p	the probability that both advertisers present in the consumers' consideration set
P_i $(i = A, B)$	the price that each advertiser charges

3.3 Equilibrium in the Price-Setting Game

Summarizing the cases when only one advertiser or both of them are in the consideration set, the total demand for each advertiser is given by $D_i = p_i D_i^1 + pD_i^2$ (Sen, 2005), where $i = A, B,$ which gives

$$\left.\begin{aligned} D_A &= p_A \times 1 + p \times D_A^2 = p_A + \frac{p}{2}(P_B - P_A + 1) \\ D_B &= p_B \times 1 + p \times D_B^2 = p_B + \frac{p}{2}(P_A - P_B + 1) \end{aligned}\right\}. \tag{3.3.1}$$

Each advertiser establishes a price, denoted by upper-case P_i, that maximizes the profit given the price of the other advertiser. The profit functions for the two advertisers are the following:

$$\left.\begin{aligned} \Pi_A &= \left[p_A + \frac{p}{2}(P_B - P_A + 1) \right] P_A - c_A \\ \Pi_B &= \left[p_B + \frac{p}{2}(P_A - P_B + 1) \right] P_B - c_B \end{aligned}\right\}, \tag{3.3.2}$$

where c_i $(i = A,B)$ is a cost of the SEM strategy.

The profit-maximizing prices are determined by the solutions of the equations system, where first order conditions of the profit functions are equated to zero. Thus, the first order conditions of the profit functions form the following system of equations:

$$\left.\begin{aligned} P_A &= \frac{p_A}{p} + \frac{P_B + 1}{2} \\ P_B &= \frac{p_B}{p} + \frac{1}{2}(P_A + 1) \end{aligned}\right\}.$$

Solving for P_A and P_B, gives

$$\left.\begin{aligned} P_A^* &= 1 + \frac{2}{3p}(2p_A + p_B) \\ P_B^* &= 1 + \frac{2}{3p}(p_A + 2p_B) \end{aligned}\right\}. \tag{3.3.3}$$

Inserting of the obtained competitive prices (3.3.3) into (3.3.1) and (3.3.2) gives equilibrium demand and profits of each advertiser. The results are presented in Table 1.

Table 1. Equilibrium statistics

Advertiser	Price	Demand	Profit
A	$P_A^* = 1 + \frac{2}{3p}(2p_A + p_B)$	$D_A^* = \frac{1}{2}p + \frac{1}{3}(2p_A + p_B)$	$\Pi_A^* = \frac{1}{2}p + \frac{2}{3}(2p_A + p_B) + \frac{2}{9p}(2p_A + p_B)^2 - c_A$
B	$P_B^* = 1 + \frac{2}{3p}(p_A + 2p_B)$	$D_B^* = \frac{1}{2}p + \frac{1}{3}(p_A + 2p_B)$	$\Pi_B^* = \frac{1}{2}p + \frac{2}{3}(p_A + 2p_B) + \frac{2}{9p}(p_A + 2p_B)^2 - c_B$

Notes: $c_A = c_B = c$

From Table 1, it is easy to see that the profits of each advertiser depend on the probabilities p, p_A, and p_B, where the probability that only one advertiser will appear in the consideration set (p_A, p_B), as well the probability that both advertisers will be viewed by the consumers (p), depend on the search engine marketing strategy that each advertiser has decided to choose. Hence, given the SEM strategy, all the probabilities are computed in Table 2.

Table 2. Probabilities associated to different SEM strategies

SEM strategy of advertiser B	*SEM strategy of advertiser A*			
	No SEM strategy	SEO	PP	SEO+PP
No SEM strategy	$p = \alpha\sigma \times \alpha\sigma = \alpha^2\sigma^2$ $p_A = \alpha\sigma(1-\alpha\sigma)$ $p_B = \alpha\sigma(1-\alpha\sigma)$	$p = \alpha\sigma \times (1 \times \sigma) = \alpha\sigma^2$ $p_A = \sigma(1-\alpha\sigma)$ $p_B = \alpha\sigma(1-\sigma)$	$p = \alpha^2\sigma^2$ $p_A = (1-\alpha\sigma)^2 + \alpha\sigma(1-\alpha\sigma)$ $p_B = \alpha\sigma(1-\alpha\sigma)$	$p = \alpha\sigma^2$ $p_A = \sigma(1-\alpha\sigma) + (1-\alpha\sigma)(1-\sigma)$ $p_B = \alpha\sigma(1-\sigma)$
SEO	$p = \alpha\sigma^2$ $p_A = \alpha\sigma(1-\sigma)$ $p_B = \sigma(1-\alpha\sigma)$	$p = \sigma^2$ $p_A = \sigma(1-\sigma)$ $p_B = \sigma(1-\sigma)$	$p = \alpha\sigma^2$ $p_A = (1-\alpha\sigma)(1-\sigma) + \alpha\sigma(1-\sigma)$ $p_B = \sigma(1-\alpha\sigma)$	$p = \sigma^2$ $p_A = (1-\sigma)^2 + \sigma(1-\sigma)$ $p_B = \sigma(1-\sigma)$

SEM strategy of advertiser B	*SEM strategy of advertiser A*			
	No SEM strategy	SEO	PP	SEO+PP
PP	$p=\alpha^2\sigma^2$ $p_A=\alpha\sigma(1-\alpha\sigma)$ $p_B=(1-\alpha\sigma)^2+$ $\alpha\sigma(1-\alpha\sigma)$	$p=\alpha\sigma^2$ $p_A=\sigma(1-\alpha\sigma)$ $p_B=(1-\alpha\sigma)(1-\sigma)$ $+\alpha\sigma(1-\sigma)$	$p=(1-\alpha\sigma)^2+\alpha^2\sigma^2$ $p_A=\alpha\sigma(1-\alpha\sigma)$ $p_B=\alpha\sigma(1-\alpha\sigma)$	$p=\alpha\sigma^2$ $+(1-\alpha\sigma)(1-\sigma)$ $p_A=\sigma(1-\alpha\sigma)$ $p_B=\alpha\sigma(1-\sigma)$
SEO+PP	$p=\alpha\sigma^2$ $p_A=\alpha\sigma(1-\sigma)$ $p_B=\sigma(1-\alpha\sigma)$ $+(1-\alpha\sigma)(1-\sigma)$	$p=\sigma^2$ $p_A=\sigma(1-\sigma)$ $p_B=(1-\sigma)^2$ $+\sigma(1-\sigma)$	$p=\alpha\sigma^2$ $+(1-\alpha\sigma)(1-\sigma)$ $p_A=\alpha\sigma(1-\sigma)$ $p_B=\sigma(1-\alpha\sigma)$	$p=(1-\sigma)^2+\sigma^2$ $p_A=\sigma(1-\sigma)$ $p_B=\sigma(1-\sigma)$

Notes: the strategies "not to invest in SEM", "invest in SEO", "PP" or "in both SEO and PP" are denoted by "No SEM strategy", "SEO", "PP" and "SEO+PP" respectively.

When only one advertiser invests in SEO, the probability that only this advertiser will be in the consideration set is $p_i=\alpha\sigma(1-\alpha\sigma)=(1\times\sigma)(1-\alpha\sigma)=\sigma(1-\alpha\sigma)$. When both advertisers invest in SEO, the probability that only one will be in the consideration set is $p_i=\sigma(1-\sigma)$. When only one advertiser invests in PP, the probability that only this advertiser will be in the consideration set is $p_i=(1-\alpha\sigma)^2+\alpha\sigma(1-\alpha\sigma)$. When both advertisers invest in PP, the probability that both of them will be in the consideration set is $p=(1-\alpha\sigma)^2+\alpha^2\sigma^2$; and so on.

Substitution of the obtained values for p, p_A and p_B from Table 2 into Table 1 gives the values of the equilibrium profits that are calculated in Table 3.

Table 3. Profit functions according to the various SEM scenarios

SEM strategy of advertiser B	*SEM strategy of advertiser A*			
	No SEM strategy	SEO	PP	SEO+PP
No SEM strategy	$\Pi_A^N = \Pi_B^N = \frac{1}{2}(-2+\alpha\sigma)^2$	$\Pi_A^{SEO} = \frac{(-4+\alpha(-2+3\sigma))^2-18\alpha c}{18\alpha}$ $\Pi_B^N = \frac{(-2+\alpha(-4+3\sigma))^2}{18\alpha}$	$\Pi_A^{PP} = \frac{(4-2\alpha\sigma+\alpha^2\sigma^2)^2-18\alpha^2\sigma^2 c}{18\alpha^2\sigma^2}$ $\Pi_B^N = \frac{(2+2\alpha\sigma-\alpha^2\sigma^2)^2}{18\alpha^2\sigma^2}$	$\Pi_A^{SEO+PP} = \frac{(4+\alpha(-2+\sigma)\sigma)^2-36\alpha\sigma^2 c}{18\alpha\sigma^2}$ $\Pi_B^N = \frac{(-2+\alpha(-2+\sigma)\sigma)^2}{18\alpha\sigma^2}$
SEO	$\Pi_A^N = \frac{(-2+\alpha(-4+3\sigma))^2}{18\alpha}$ $\Pi_B^{SEO} = \frac{(-4+\alpha(-2+3\sigma))^2-18\alpha c}{18\alpha}$	$\Pi_A^{SEO} = \frac{1}{2}((-2+\sigma)^2-2c)$ $\Pi_B^{SEO} = \frac{1}{2}((-2+\sigma)^2-2c)$	$\Pi_A^{PP} = \frac{(4-2\sigma+\alpha\sigma^2)^2-18\alpha\sigma^2 c}{18\alpha\sigma^2}$ $\Pi_B^{SEO} = \frac{(2+2\sigma-\alpha\sigma^2)^2-18\alpha\sigma^2 c}{18\alpha\sigma^2}$	$\Pi_A^{SEO+PP} = \frac{(4-2\sigma+\sigma^2)^2-36\sigma^2 c}{18\sigma^2}$ $\Pi_B^{SEO} = \frac{(-2-2\sigma+\sigma^2)^2-18\sigma^2 c}{18\sigma^2}$
PP	$\Pi_A^N = \frac{(2+2\alpha\sigma-\alpha^2\sigma^2)^2}{18\alpha^2\sigma^2}$ $\Pi_B^{PP} = \frac{(4-2\alpha\sigma+\alpha^2\sigma^2)^2-18\alpha^2\sigma^2 c}{18\alpha^2\sigma^2}$	$\Pi_A^{SEO} = \frac{(2+2\sigma-\alpha\sigma^2)^2-18\alpha\sigma^2 c}{18\alpha\sigma^2}$ $\Pi_B^{PP} = \frac{(4-2\sigma+\alpha\sigma^2)^2-18\alpha\sigma^2 c}{18\alpha\sigma^2}$	$\Pi_A^{PP} = \frac{1+(-2+4\alpha\sigma-4\alpha^2\sigma^2)c}{2-4\alpha\sigma+4\alpha^2\sigma^2}$ $\Pi_B^{PP} = \frac{1+(-2+4\alpha\sigma-4\alpha^2\sigma^2)c}{2-4\alpha\sigma+4\alpha^2\sigma^2}$	$\Pi_A^{SEO+PP} = \frac{(-3+(-1+\alpha)\sigma)^2}{18(1-(1+\alpha)\sigma+2\alpha\sigma^2)}-2c$ $\Pi_B^{PP} = \frac{(3+(-1+\alpha)\sigma)^2}{18(1-(1+\alpha)\sigma+2\alpha\sigma^2)}-c$
SEO+PP	$\Pi_A^N = \frac{(-2+\alpha(-2+\sigma)\sigma)^2}{18\alpha\sigma^2}$ $\Pi_B^{SEO+PP} = \frac{(4+\alpha(-2+\sigma)\sigma)^2-36\alpha\sigma^2 c}{18\alpha\sigma^2}$	$\Pi_A^{SEO} = \frac{(-2-2\sigma+\sigma^2)^2-18\sigma^2 c}{18\sigma^2}$ $\Pi_B^{SEO+PP} = \frac{(4-2\sigma+\sigma^2)^2-36\sigma^2 c}{18\sigma^2}$	$\Pi_A^{PP} = \frac{(3+(-1+\alpha)\sigma)^2}{18(1-(1+\alpha)\sigma+2\alpha\sigma^2)}-c$ $\Pi_B^{SEO+PP} = \frac{(-3+(-1+\alpha)\sigma)^2}{18(1-(1+\alpha)\sigma+2\alpha\sigma^2)}-2c$	$\Pi_A^{SEO+PP} = \frac{1+(-4+8\sigma-8\sigma^2)c}{2-4\sigma+4\sigma^2}$ $\Pi_B^{SEO+PP} = \frac{1+(-4+8\sigma-8\sigma^2)c}{2-4\sigma+4\sigma^2}$

Notes: Π_i^N indicates equilibrium profit function for advertiser *i* (*i* = *A*, *B*) when no SEM strategy is implemented. Π_i^{SEO} corresponds to the profit function for advertiser *i* when SEO strategy is implemented. Π_i^{PP} and Π_i^{SEO+PP} indicate profit functions for advertiser *i* when PP strategy and strategy to invest in both PP and SEO respectively are used. It is assumed that $c_A = c_B = c$. When advertiser A or B doesn't implement any SEM strategy, $c_i = 0$. When implemented SEM strategy implies investing both in SEO and PP, an advertiser *i* incurs cost of 2*c*.[12]

[12] Despite Sen (2005) in his research study doesn't consider a SEM strategy to invest in both SEO and PP as a potential optimal strategy for advertiser A or B, in our study we include this SEM strategy in the further analysis since, when the SE's manipulation is available, the probability $\alpha\sigma$ is no more equal 1. In Sen (2005) the strategy to invest in both SEO and PP implies incurring the double cost 2*c* and the effect that is the same as from investing only in SEO, where $\alpha = 1$ if the SE's manipulation is not available.

3.4 Optimal search engine marketing strategy

Various combinations of two advertisers' SEM strategies give different profit functions. The comparison of such profit functions allows forming the following propositions:

Proposition 1: When the cost of investing in either in SEO or PP, or both in SEO and PP is higher than the cost constraints (*see proof to Proposition 1*), i.e. $c_i > C1, C2$ and $C3$ (where i = A,B; and *C1*, *C2* and *C3* are given in Figure 1), given any value of σ, both advertisers are better off not choosing any SEM strategy (see Figure 1).

When the values of α are relatively high, even if cost c_i (where i = A,B) is relatively small but higher than *C1*, *C2* and *C3* (see Figure 1), then no advertiser invests in any SEM strategy. However, if the values of α are relatively low, and advertisers face the high probability not to be listed in the top of organic search results, they will prefer to invest in some SEM strategy even if cost c_i is relatively high (given that $c_i < C1, C2$ and $C3$).

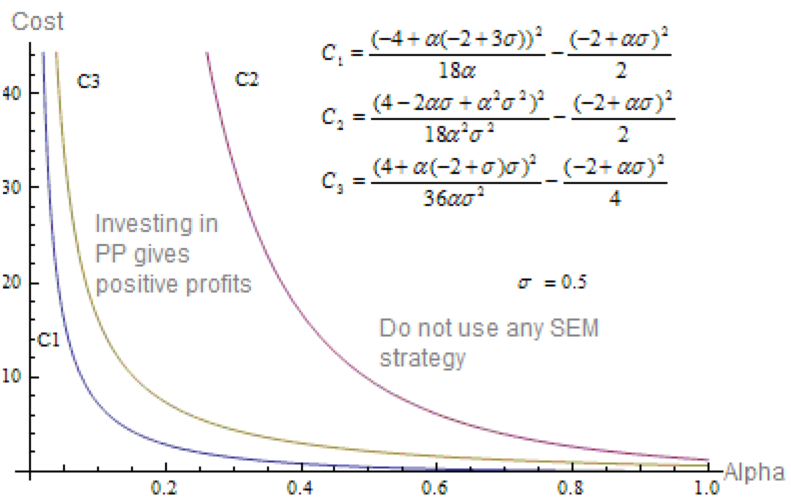

Figure 1. Cost constraints for a choice of the SEM strategy

Proof of the Proposition 1

Assuming that one advertiser chooses not to invest in any SEM strategy, the other advertiser will not also invest in any SEM strategy if the cost c_i for implementing each strategy will be higher than cost constraints. The cost constraints can be found from the following inequalities:

$$\Pi_i^N > \Pi_i^{SEO}, \Pi_i^N > \Pi_i^{PP}, \Pi_i^N > \Pi_i^{SEO+PP} (i = A, B).$$

When $\Pi_i^N > \Pi_i^{SEO}$,

$$\frac{1}{2}(-2+\alpha\sigma)^2 > \frac{(-4+\alpha(-2+3\sigma))^2 - 18\alpha c_i}{18\alpha}$$

$$c_i > \frac{(-4+\alpha(-2+3\sigma))^2}{18\alpha} - \frac{(-2+\alpha\sigma)^2}{2}$$

$$c_i > C1, \text{ where } C1 = \frac{(-4+\alpha(-2+3\sigma))^2}{18\alpha} - \frac{(-2+\alpha\sigma)^2}{2}.$$

When $\Pi_i^N > \Pi_i^{PP}$,

$$\frac{1}{2}(-2+\alpha\sigma)^2 > \frac{(4-2\alpha\sigma+\alpha^2\sigma^2)^2 - 18\alpha^2\sigma^2 c_i}{18\alpha^2\sigma^2}$$

$$c_i > \frac{(4-2\alpha\sigma+\alpha^2\sigma^2)^2}{18\alpha^2\sigma^2} - \frac{(-2+\alpha\sigma)^2}{2}$$

$$c_i > C2, \text{ where } C2 = \frac{(4-2\alpha\sigma+\alpha^2\sigma^2)^2}{18\alpha^2\sigma^2} - \frac{(-2+\alpha\sigma)^2}{2}.$$

When $\Pi_i^N > \Pi_i^{SEO+PP}$,

$$\frac{1}{2}(-2+\alpha\sigma)^2 > \frac{(4+\alpha(-2+\sigma)\sigma)^2 - 36\alpha\sigma^2 c_i}{18\alpha\sigma^2}$$

$$c_i > \frac{(4+\alpha(-2+\sigma)\sigma)^2}{36\alpha\sigma^2} - \frac{(-2+\alpha\sigma)^2}{4}$$

$$c_i > C3, \text{ where } C3 = \frac{(4+\alpha(-2+\sigma)\sigma)^2}{36\alpha\sigma^2} - \frac{(-2+\alpha\sigma)^2}{4}.$$

In the Figure 1 it is easy to see that the area above the line of C2 corresponds to the situation when no advertiser invests in any SEM strategy since $c_i > C1, C2$ and $C3$. The area above the line of *C3* and below *C2* corresponds to the case when investing in paid placement gives positive profits for advertiser A or B, the area above the line of *C1* and below *C3* indicates the situation when investing in both SEO and PP, as well as only in PP, gives positive profits. Finally, the area below *C1* corresponds to the positive profits by investing in SEO, PP or both in SEO and PP.

In the case presented in the Figure 1, it is assumed that the probability that the SE's ranking algorithm is of quality, σ, is equal to 0.5. Simulations of the scenarios when $\sigma >$ 0.5 and $\sigma <$ 0.5 are presented in the Appendix A[13]. Therefore, according to the Appendix A, it is possible to see that when the cost of investing in either in SEO or PP, or both in

[13] According to the different values of parameter σ, condition C2 > C3 > C1 holds when σ < 0.7. When σ > 0.7, the inequality is C 2> C1> C3.

SEO and PP is such that $c_i > C1, C2$ and $C3$ (where i = A,B; and *C1*, *C2* and *C3* are given in Figure 1), given any value of σ, both advertisers are better off not choosing any SEM strategy.

Proposition 2: There exists a threshold value α^* such that when $\alpha \leq \alpha^*$ and $c_i < C1, C2$ and $C3$ (where i = A,B; and *C1*, *C2* and *C3* are given in the Figure 1), given any value of σ, the pure strategy Nash equilibrium is defined when one advertiser invests in PP and another doesn't implement any SEM strategy (see Figure 2 and Figure 3).

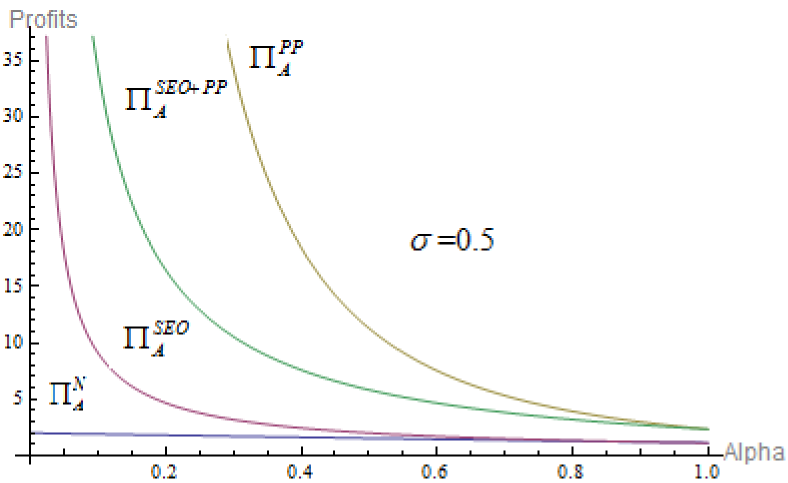

Figure 2. When advertiser B doesn't invest in any SEM strategy, advertiser A's strategy to invest in PP weakly dominates the strategy to invest in both SEO and PP, and strictly dominates all other strategies, given $\sigma = 0.5$.

Proof of the Proposition 2

When $c_i < C1, C2$ and $C3$, without loss of generality, it is assumed that advertiser B doesn't invest in any SEM strategy. Then, advertiser A responses by investing in the paid placement since $\Pi_A^{PP} > \Pi_A^{SEO+PP} > \Pi_A^{SEO} > \Pi_A^N$ (see Figure 2). In the Figure 2 it is easy to see that A's strategy to invest in PP weakly dominates other strategies because, as α approaches 1, the strategies to invest in PP and to invest in both SEO and PP coincide, given the value of σ equal 0.5. Analyzing the strategies for advertiser A when σ > 0.5 and when σ < 0.5 (Appendix B), it is easy to see that, as σ approaches 1, all advertiser A's possible profits coincide for relatively higher value of α. Thus, when $\sigma = 0.98$, according to all possible SEM strategies, A's profits are the same when $\alpha \geq 0.8$.[14]

When advertiser A decides to invest in the paid placement, advertiser B's best response is to do nothing when $\alpha \leq \alpha^*$ (see Figure 3), given $\sigma = 0.5$.[15] Thus, the pure strategy Nash equilibrium is defined when advertiser A invests in the paid placement and advertiser B doesn't invest in any SEM strategy.

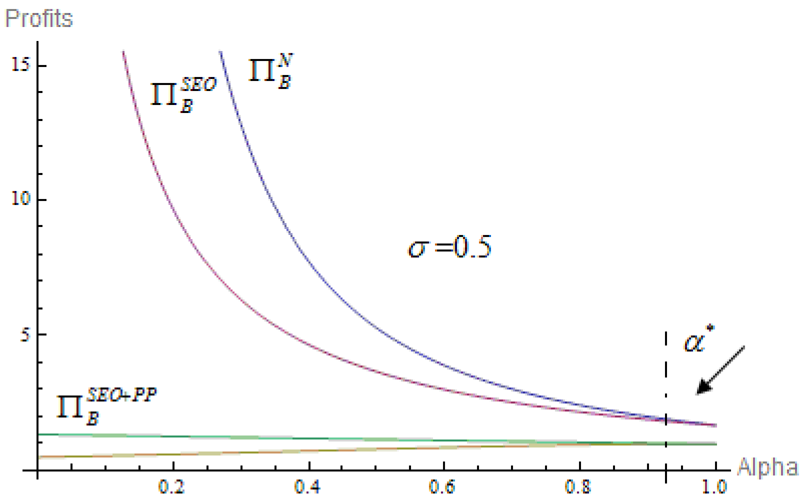

Figure 3. When advertiser A chooses the strategy to invest in PP, Advertiser B's best response is not to use any SEM strategy when $\alpha \leq \alpha^*$.

Another way to find the pure strategy Nash equilibrium in this *symmetric game* is to build a payoff matrix and define which advertisers' strategies are mutually best responses (see Table 4).

From the Table 4 it is easy to see that both advertisers get the maximum payoff when one of them invests in the paid placement and other doesn't make any investments in search engine marketing.

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[14] When the SE doesn't manipulate and keeps a high value of σ, as the advertiser's probability to be in the consideration set, related to the quality of her website, α approaches 1, the advertiser becomes indifferent in which SEM strategy to invest. Thus, the SE has a strong incentive to lower σ in order to avoid the advertiser's strategy not to invest in the paid placement.

[15] Similarly, as σ approaches 1, all advertiser B's payoffs coincide for relatively higher value of α^*. Thus, when $\sigma = 0.98$, B's profits are the same according to all possible SEM strategies when $\alpha^* \geq 0.75$ (Appendix B).

Table 4. Advertisers' Payoff Matrix

Advertiser B	Advertiser A			
SEM strategy	*Do nothing*	*Invest in SEO*	*Invest in PP*	*Invest in both SEO and PP*
Do nothing	1.53 1.53	2.00 1.17	11.27 5.28	5.82 2.51
Invest in SEO	1.17 2.00	1.12 1.12	4.33 3.66	2.33 1.67
Invest in PP	5.28 11.27	3.66 4.33	0.79 0.79	1.15 1.16
Invest in SEO and PP	2.51 5.82	1.67 2.33	1.16 1.15	0.98 0.98

The presented numbers are calculated taking into account cost constraints. It was assumed that $\alpha = 0.5, \sigma = 0.5, c_i = 0.01$. The first number in the cell corresponds to the A's payoff, the second one corresponds to the B's payoff. In the Appendix C there are presented Advertisers' Payoff Matrixes, calculated on the basis of different values of probabilities α and σ, fixing constrained cost $c_i = 0.01$.

3.5 Discussion and limitations of the model

In the previous section there have been studied the search engine marketing strategies of two advertisers in the symmetric game. The main result of the performed analysis shows that the optimal SEM strategy for one advertiser is to invest in the paid placement whereas the optimal strategy for other one is to do nothing. Mutually best responses of two advertisers form the pure strategy Nash equilibrium in the symmetric game.

Why it is optimal for one advertiser invests in the paid placement given that other doesn't choose any SEM strategy?

To answer this question, it is enough to look on the equilibrium prices and demand for both advertisers and to compare the indicators according to different SEM strategies. Since the analysis of advertisers' strategies show that the choice of the optimal strategy doesn't significantly depend on the value of the probabilities α and σ, assuming that $\alpha = 0.5$ and

$\sigma = 0.5$, it becomes possible to calculate the equilibrium price and demand for each advertiser.

Hence, if advertiser B doesn't choose any SEM strategy, she sets a price equal to 13 and has a market share of 0.4. In turn, advertiser A, who invests in PP, sets a price equal to 19 and has a market share of 0.6. Clearly, being initially identical with advertiser B, but investing in the paid placement, advertiser A has an advantage of more likely appearance in the consumers' consideration set. Hence, the probability that only advertiser A will be viewed by the consumers (that corresponds to the case of a monopoly on a market) is higher than one of advertiser B. Since the demand directly depends on this probability, as a result, a market share of A is higher than one of B. It allows A to charge a higher price and end up with higher profits.

Further calculations show that, when both advertisers implement the same SEM strategy (SEO, PP or SEO+PP), they both end up with much lower profits. When both advertisers invest in the search engine online marketing, the probability that only one advertiser will be viewed by the consumers (probability to be a monopolist) is lower while the probability that both advertisers appear in the top of the organic listing is higher. Hence, to get larger market share, advertisers start lowering their prices. That's why, when advertiser A invests in PP, advertiser B, investing also in PP, sets an equilibrium price equal to 1.6, but has a market share of 0.5. Clearly, such a sharp decrease in the B's equilibrium price (from 13 to 1.6) can not bring more revenues even with an increase in a market share from 0.4 to 0.5. Moreover, implementing some SEM strategy, B occurs cost of *c* or *2c*. In this situation, advertiser A also forces to lower her price in order not to loose the position in the market. Therefore, when both advertisers invest in PP, they share equally the market, but have the possible minimum profits of 0.79 (Table 4).[16]

[16] The case when both advertisers that use the same SEM strategy and, being identical, start to compete in prices to get larger market share, can seem to be similar to the Bertrand competition with the weak Nash-equilibrium when equilibrium prices are equal to marginal cost (Dinesh et al., 2009). However, in our study, following Hotelling (1929), we find that if an advertiser increases her price, she will gradually lose business to her rivals, but she will not lose all her market share at once when she raises her price only a bit. Hotelling (1929) claims that "the assumption that all buyers deal with the cheapest seller leads to a type of instability which disappears when the quantity sold by each is considered as a continuous function of the differences in price. The use of such a continuous function does, to be sure, seem to violate the doctrine that in one market there can at one time be only one price. But this doctrine is only valid when the commodity in question is absolutely standardised in all respects and when the "market" is a point, without length, breadth or thickness."

Another possible explanation why an advertiser decides to response by not investing in a SEM strategy, given that other invests in PP, is the specificity of the product she boosts. Thus, if after entering related keywords, only few relevant web sites that sell similar products are displayed, an advertiser is better off not to waste money, investing in SEM. Even with the relatively low probability to be located in the top of the organic listing, an advertiser will be still confident that her web site will be found by the consumers even as the 'back pages' of the organic search results. Certainly, having few alternatives, consumers will try to view all the relevant websites and compare purchase terms. The consumer will definitely choose an advertiser that offers the lowest 'discomfort cost' (free shipping, appropriate conditions of product returns, etc.).

Does the SE's manipulation of organic search results matters in the advertisers' choices of SEM strategies?

As it has been mentioned above, similar analysis of advertisers' strategies has been conducted by Sen (2005). Contrary to our study, he doesn't consider how SE's manipulation affects the optimal strategies of advertisers, assuming 'fair game' of the SE. Thus, Sen finds that there exists no pure strategy Nash equilibrium when $\alpha \leq \alpha^*$. Only for relatively high values of α ($\alpha > \alpha^*$), the Nash equilibrium is defined when both advertisers invest in the paid placement. Therefore, the SE is better off and gets the maximum revenue of 2*c* only when the probability of both advertisers to be listed in the top of the organic search results is rather high, $\alpha > \alpha^*$.

Since, in the Sen's study, the SE can not artificially increase α to get the highest profits, it is always worse off when α is relatively low. Consequently, having the dominant position, the SE has an incentive to manipulate the probability of advertisers to be listed in the top of the organic search results. Modifying artificially the efficiency of its ranking algorithm, the SE can influence the advertisers' likelihood to be displayed in the top of organic listing. Thus, as it has been described in the first section, the probability to be in the consumers' consideration set is $\alpha\sigma$, related not only to the high quality of the advertiser's relevant content (information, product, etc.), but also to the high quality of the SE's ranking algorithm.

Thereby, having introduced in the Sen's model the SE's manipulation of the organic search, we have analysed the optimal strategies of the advertisers in these new conditions.

We find that, when α is relatively low ($\alpha \leq \alpha^*$), one advertiser invests in the paid placement, bringing revenue of *c* to the SE. Thus, imposing relatively low value of σ and fixing cost for PP below constraints, the SE can assure the revenue of *c*. We can conclude that, when the SE lowers the value of σ that decreases the probability of the advertisers to be listed in the top on the organic side; advertisers end up with the lower demand. When $\alpha \leq \alpha^*$, it pushes one of the advertisers to increase her chances to be viewed by the consumers by investing in the paid placement. When $\alpha > \alpha^*$, the SE doesn't have any more a power to influence the advertisers' behaviour since they are indifferent between several SEM strategies; and it is rather difficult to find the equilibrium (Appendix B).

In their study, Katona and Sarvary (2010) show that it can be a situation when for the SE it will be optimal to decrease the amount of sponsored links it offers. Certainly, more sponsored links imply more payments from the advertisers; however, when the number of such links is too high, the traffic per each link goes down. Hence, if the SE decreases the number of sponsored links, the traffic increases, and advertisers are willing to pay more for the paid placement. Therefore, even when only one advertiser purchases a sponsored link, the SE's manipulation costs are offset by the higher traffic, flowing through the advertiser's website.

Limitations of the model

Since the proposed model is rather simple, being based on many assumptions, it has certain limitations. Thus, the model implies the existence of the monopolistic search engine that doesn't reflect the reality where the leading online SE Google possesses around 70% of all the searches (comScore, March 2013 U.S.). Apart from this, the model describes the strategies of only two players (advertisers A and B) in the static symmetric game. It can not explain how the advertisers' strategies could change over time.

In addition, it is assumed that the cost of implementing SEO and paid placement are the same. Certainly, in reality, the costs of different SEM strategies are explained by many factors, and they are different across markets. Thus, how much each advertiser will spend on SEO depends on the initial content quality of the advertiser's website, number of SEO tools are planned to be implemented by the SEO company, amount of content to be improved, etc. In turn, the cost of the paid placement mostly depends on the popularity of inserted keywords, the cost per click, the amount of performed clicks (according to Pay-

Per-Click pricing model), and on the period of displaying the link in the sponsored listing. Nevertheless, experts emphasize that the cost to implement SEO programmes for several hundred keywords is much higher than the purchase of sponsored links for campaigns that include thousands of keywords. That's why the market share of SEO is considerably smaller than of PP.[17]

In spite of the mentioned limitations, the model is rather realistic to explain the logic of the online marketing functioning and the rationality of the advertisers' choice of a definite SEM strategy in the conditions of the organic search results manipulation.

3.6 Further discussion and conclusions

Further discussion

Relaxation of the assumption about constrained costs

The model can be extended to allow cost c for implementing SEM strategy being no more below cost constraints C1, C2 and C3. The intuition behind the relaxation of the constrained cost assumption lies in the fact that the SE always has an incentive to increase its payoff. It can gradually raise the cost of the paid placement till the moment while the equilibrium still exists and it is still optimal for one advertiser to invest in PP. Thus, the future analysis can be conducted to indentify the threshold value of such cost, c^*, where equilibrium exists while $c \le c^*$. The one limitation of such analysis is that, assuming higher value of c as the cost for implementing PP, it is also necessary to assume higher value of c as the cost for implementing SEO, because they are initially equal according to the specificity of the model.

Consumers' payoff when SE's manipulation is available or not

Since, in this dissertation work, consumers' benefits from the purchasing on the advertisers' websites have not been analysed, it can be done as a part of the further study. Thus, consumers' benefits can be compared when SE's manipulation of the organic search

[17] Grieselhuber, R. 2012. 2012 SEO & Inbound Marketing Outlook. GinzaMetrics. http://www.ginzametrics.com/blog/2012-seo-and-inbound-marketing-outlook

results is available or not. Clearly, when the SE decreases the probability of the advertiser to be viewed by the visitors, consumers' payoff is lower. The only way to offset such decrease is to lower 'discomfort consumers' cost' by the advertisers.

Relaxation of the assumption about costless provision of SE's manipulation

In this study, it is assumed that, implementing manipulation of the organic search results, the SE doesn't incur additional costs. In reality, search engines have to make investments in the update of their ranking algorithms. Evidently, it decreases SEs' profits. Thus, one can extend the model, including the cost of SE's manipulation to get more accurate SE's profit function.

Conclusions

The right choice of the search engine marketing strategy is very important in online marketing. Investing in search engine optimization helps to improve the organic ranking of the website in the search results, whereas investing in paid placement gives an advertiser's website additional chance to be visited if she is not in the consumers' consideration set in the organic listing. Both strategies have their own advantages and disadvantages. Thus, it can seem not very rational for an advertiser to invest in the paid placement, given that consumers always start their search on the organic side and switch on the sponsored side only if they are not satisfied (Jerath et al.,2013).

Contrariwise, investing in SEO is more costly than the purchase of the paid placement. Moreover, different search engines have different requirements about search engine optimization, white hat SEO. If the page is correctly optimized for Google and gets a high rank, it doesn't mean that it will get also a high rank on Yahoo, which has its own SEO regulation. Thus, it becomes rather difficult to predict how successful SEO programme will be, whether the website gets the high rank in the organic search results or not. The situation is even more complicated when the SE starts to artificially decrease the organic rank of advertisers. Clearly, SEO becomes less efficient in this situation.

On the contrary, investment in PP gives a guarantee that the website will be visited at least on the sponsored site. In the conditions of SE's manipulation of the organic search results, the probability that the website will be visited on the sponsored side is even higher since the SE decreases the probability of the website to be in a top of the organic listing. Therefore, facing the downward pressure on the organic rank, it will be optimal at least for one advertiser to purchase paid placements.

4 Conclusions

We believe that the economics of online search is a subject of high importance to both academics and web managers. According to the research study conducted by IAB, 426.9 million Europeans are online every week. An average European spends around 15 hours for usage of the Internet each week. Moreover, 96% of European internet users search online for goods and information (Jonas Koponen, V Intertic Conference on Antitrust Policy, 2013). To find what they look for, Europeans visit both organic and paid links. Consumers are better off when such organic and paid links are of high quality.

The topic of the organic search results quality in online advertising is new and not widely investigated yet. In our study, we analyse the role of search engine optimization and SE's manipulation in organic search quality modification. We explain when and why a SE can have an incentive to decrease its organic search quality through its ranking algorithm. In order to determine an impact of SE's manipulation on the advertisers' strategies, we build the model that assumes the existence of the monopolistic SE, two identical advertisers and consumers who find optimal to start online search by visiting organic links first.

The main result of the conducted analysis shows that in the conditions of SE's decrease in its organic links quality, the optimal strategy for one advertiser is to invest in sponsored links whereas for other one is to use no SEM strategy. The role of search engine optimization is neglected in this study since no advertiser chooses to implement white hat SEO and increase her website's content relevance. Given the complexity of the problem, our model has several limitations, but it is still rather realistic. However, we leave the issue of consumer welfare in the conditions of SE's manipulation for the further study. The relaxation of several assumptions could also be explored by future research.

In this graduate dissertation, we shed light on the process of organic search, search engine optimization and SE's assigning of the scores to the websites through the ranking algorithm. We explain possible online search engines', advertisers' and consumers' strategies according to different scenarios. Particular attention is devoted to the modelling of the situation when the leading online search engine tries to offset the negative affect of SEO on the SE's revenues and decreases the algorithm quality to lower the organic rank of high quality websites.

5 Appendices

Appendix A

Wolfram Mathematica code to build Figure 1

```
Subscript[c,1]=((-4+α*(-2+3*σ))^2)/(18*α)-((-2+α*σ)^2)/2
Subscript[c,2]=((4-2*α*σ+α*α*σ*σ)^2)/(18*α*α*σ*σ)-((-2+α*σ)^2)/2
Subscript[c,3]=((4+α*(-2+σ)*σ)^2)/(36*α*σ*σ)-((-2+α*σ)^2)/4

Plot[{((-4+α*(-2+3*0.5))^2)/(18*α)-((-2+α*0.5)^2)/2,((4-
2*α*0.5+α*α*0.5*0.5)^2)/(18*α*α*0.5*0.5)-((-2+α*0.5)^2)/2,((4+α*(-
2+0.5)*0.5)^2)/(36*α*0.5*0.5)-((-2+α*0.5)^2)/4},{α,0,1}]

Plot[{((-4+α*(-2+3*0.1))^2)/(18*α)-((-2+α*0.1)^2)/2,((4-
2*α*0.1+α*α*0.1*0.1)^2)/(18*α*α*0.1*0.1)-((-2+α*0.1)^2)/2,((4+α*(-
2+0.1)*0.1)^2)/(36*α*0.1*0.1)-((-2+α*0.1)^2)/4},{α,0,1}]

Plot[{((-4+α*(-2+3*0.2))^2)/(18*α)-((-2+α*0.2)^2)/2,((4-
2*α*0.2+α*α*0.2*0.2)^2)/(18*α*α*0.2*0.2)-((-2+α*0.2)^2)/2,((4+α*(-
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Plot[{((-4+α*(-2+3*0.3))^2)/(18*α)-((-2+α*0.3)^2)/2,((4-
2*α*0.3+α*α*0.3*0.3)^2)/(18*α*α*0.3*0.3)-((-2+α*0.3)^2)/2,((4+α*(-
2+0.3)*0.3)^2)/(36*α*0.3*0.3)-((-2+α*0.3)^2)/4},{α,0,1}]

Plot[{((-4+α*(-2+3*0.6))^2)/(18*α)-((-2+α*0.6)^2)/2,((4-
2*α*0.6+α*α*0.6*0.6)^2)/(18*α*α*0.6*0.6)-((-2+α*0.6)^2)/2,((4+α*(-
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2+0.7)*0.7)^2)/(36*α*0.7*0.7)-((-2+α*0.7)^2)/4},{α,0,1}]

Plot[{((-4+α*(-2+3*0.8))^2)/(18*α)-((-2+α*0.8)^2)/2,((4-
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2+0.8)*0.8)^2)/(36*α*0.8*0.8)-((-2+α*0.8)^2)/4},{α,0,1}]

Plot[{((-4+α*(-2+3*0.9))^2)/(18*α)-((-2+α*0.9)^2)/2,((4-
2*α*0.9+α*α*0.9*0.9)^2)/(18*α*α*0.9*0.9)-((-2+α*0.9)^2)/2,((4+α*(-
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Plot[{((-4+α*(-2+3*0.98))^2)/(18*α)-((-2+α*0.98)^2)/2,((4-
2*α*0.98+α*α*0.98*0.98)^2)/(18*α*α*0.98*0.98)-((-2+α*0.98)^2)/2,((4+α*(-
2+0.98)*0.98)^2)/(36*α*0.98*0.98)-((-2+α*0.98)^2)/4},{α,0,1}]
```
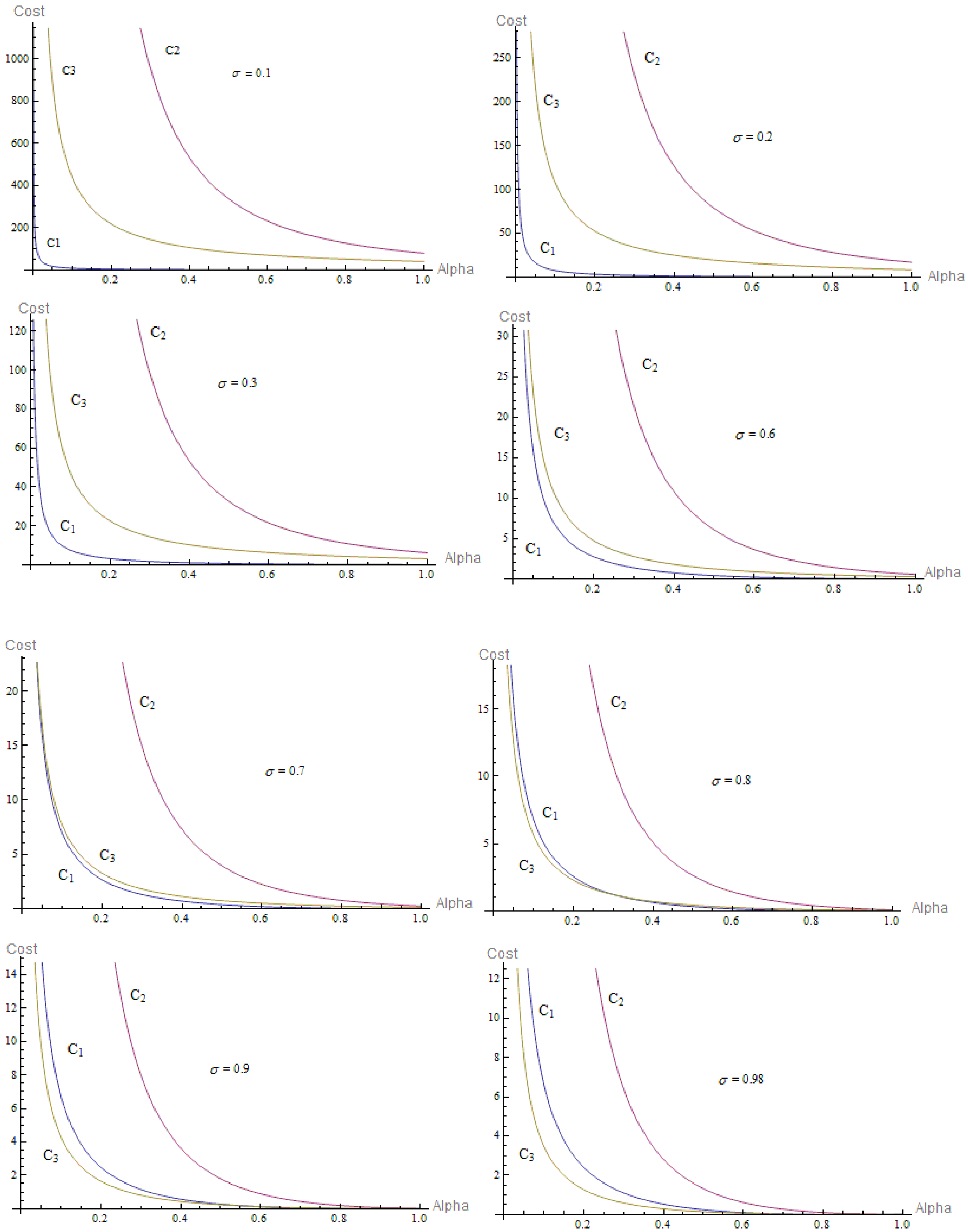

Simulations of the cost constraints depending on the value of σ



Appendix B

Wolfram Mathematica code to build Figure 2

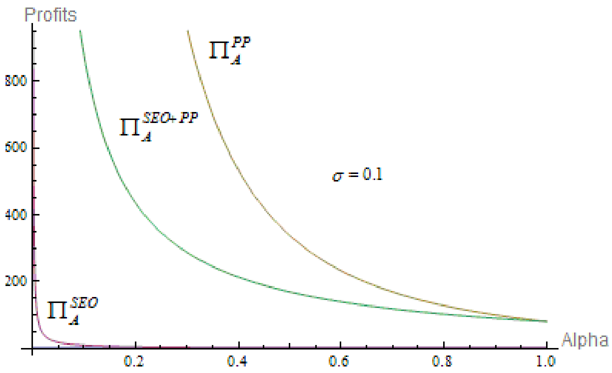
```
AProfitN=0.5*((-2+α*σ)^2)
AProfitSEO=((-4+α*(-2+3*σ))^2-18*α*c)/(18*α)
AProfitPP=((4-2*α*σ+α*α*σ*σ)^2-18*α*α*σ*σ*c)/(18*α*α*σ*σ)
AprofitSEOPP=((4+α*(-2+σ)*σ)^2-36*α*σ*σ*c)/(18*α*σ*σ)

Plot[{(0.5*((-2+α*0.5)^2)),(((-4+α*(-2+3*0.5))^2-18*α*0.01)/(18*α)),(((4-
2*α*0.5+α*α*0.5*0.5)^2-18*α*α*0.5*0.5*0.01)/(18*α*α*0.5*0.5)),(((4+α*(-
2+0.5)*0.5)^2-36*α*0.5*0.5*0.01)/(18*α*0.5*0.5))},{α,0,1}]

Plot[{(0.5*((-2+α*0.1)^2)),(((-4+α*(-2+3*0.1))^2-18*α*0.01)/(18*α)),(((4-
2*α*0.1+α*α*0.1*0.1)^2-18*α*α*0.1*0.1*0.01)/(18*α*α*0.1*0.1)),(((4+α*(-
2+0.1)*0.1)^2-36*α*0.1*0.1*0.01)/(18*α*0.1*0.1))},{α,0,1}]

Plot[{(0.5*((-2+α*0.2)^2)),(((-4+α*(-2+3*0.2))^2-18*α*0.01)/(18*α)),(((4-
2*α*0.2+α*α*0.2*0.2)^2-18*α*α*0.2*0.2*0.01)/(18*α*α*0.2*0.2)),(((4+α*(-
2+0.2)*0.2)^2-36*α*0.2*0.2*0.01)/(18*α*0.2*0.2))},{α,0,1}]

Plot[{(0.5*((-2+α*0.3)^2)),(((-4+α*(-2+3*0.3))^2-18*α*0.01)/(18*α)),(((4-
2*α*0.3+α*α*0.3*0.3)^2-18*α*α*0.3*0.3*0.01)/(18*α*α*0.3*0.3)),(((4+α*(-
2+0.3)*0.3)^2-36*α*0.3*0.3*0.01)/(18*α*0.3*0.3))},{α,0,1}]

Plot[{(0.5*((-2+α*0.4)^2)),(((-4+α*(-2+3*0.4))^2-18*α*0.01)/(18*α)),(((4-
2*α*0.4+α*α*0.4*0.4)^2-18*α*α*0.4*0.4*0.01)/(18*α*α*0.4*0.4)),(((4+α*(-
2+0.4)*0.4)^2-36*α*0.4*0.4*0.01)/(18*α*0.4*0.4))},{α,0,1}]

Plot[{(0.5*((-2+α*0.6)^2)),(((-4+α*(-2+3*0.6))^2-18*α*0.01)/(18*α)),(((4-
2*α*0.6+α*α*0.6*0.6)^2-18*α*α*0.6*0.6*0.01)/(18*α*α*0.6*0.6)),(((4+α*(-
2+0.6)*0.6)^2-36*α*0.6*0.6*0.01)/(18*α*0.6*0.6))},{α,0,1}]

Plot[{(0.5*((-2+α*0.7)^2)),(((-4+α*(-2+3*0.7))^2-18*α*0.01)/(18*α)),(((4-
2*α*0.7+α*α*0.7*0.7)^2-18*α*α*0.7*0.7*0.01)/(18*α*α*0.7*0.7)),(((4+α*(-
2+0.7)*0.7)^2-36*α*0.7*0.7*0.01)/(18*α*0.7*0.7))},{α,0,1}]

Plot[{(0.5*((-2+α*0.8)^2)),(((-4+α*(-2+3*0.8))^2-18*α*0.01)/(18*α)),(((4-
2*α*0.8+α*α*0.8*0.8)^2-18*α*α*0.8*0.8*0.01)/(18*α*α*0.8*0.8)),(((4+α*(-
2+0.8)*0.8)^2-36*α*0.8*0.8*0.01)/(18*α*0.8*0.8))},{α,0,1}]

Plot[{(0.5*((-2+α*0.9)^2)),(((-4+α*(-2+3*0.9))^2-18*α*0.01)/(18*α)),(((4-
2*α*0.9+α*α*0.9*0.9)^2-18*α*α*0.9*0.9*0.01)/(18*α*α*0.9*0.9)),(((4+α*(-
2+0.9)*0.9)^2-36*α*0.9*0.9*0.01)/(18*α*0.9*0.9))},{α,0,1}]

Plot[{(0.5*((-2+α*0.98)^2)),(((-4+α*(-2+3*0.98))^2-
18*α*0.01)/(18*α)),(((4-2*α*0.98+α*α*0.98*0.98)^2-
18*α*α*0.98*0.98*0.01)/(18*α*α*0.98*0.98)),(((4+α*(-2+0.98)*0.98)^2-
36*α*0.98*0.98*0.01)/(18*α*0.98*0.98))},{α,0,1}]
```

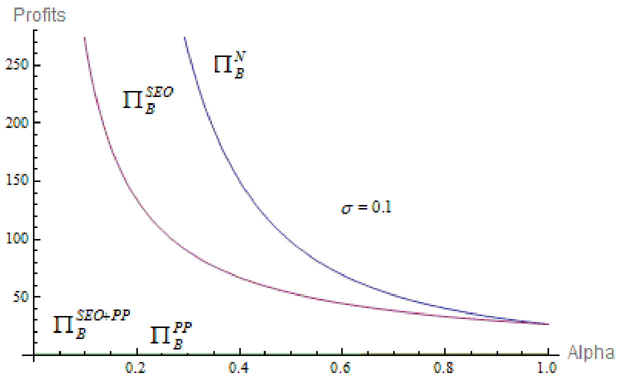
Wolfram Mathematica code to build Figure 3

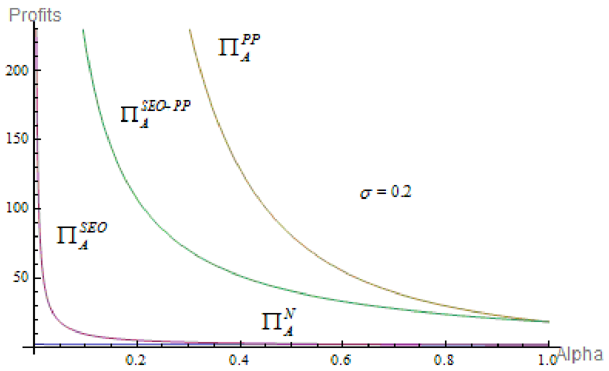
```
BProfitN=((2+2*α*σ-α*α*σ*σ)^2)/(18*α*α*σ*σ)
BProfitSEO=((2+2*σ-α*σ*σ)^2-18*α*σ*σ*c)/(18*α*σ*σ)
BProfitPP=(1+(-2+4*α*σ-4*α*α*σ*σ)*c)/(2-4*α*σ+4*α*α*σ*σ)
BProfitSEOPP=((-3+(-1+α)*σ)^2)/(18*(1-(1+α)*σ+2*α*σ*σ))-(2*c)

Plot[{(((2+2*α*0.5-α*α*0.5*0.5)^2)/(18*α*α*0.5*0.5)),(((2+2*0.5-
α*0.5*0.5)^2-18*α*0.5*0.5*0.01)/(18*α*0.5*0.5)),((1+(-2+4*α*0.5-
4*α*α*0.5*0.5)*0.01)/(2-4*α*0.5+4*α*α*0.5*0.5)),(((-3+(-
1+α)*0.5)^2)/(18*(1-(1+α)*0.5+2*α*0.5*0.5))-(2*0.01))},{α,0,1}]

Plot[{(((2+2*α*0.1-α*α*0.1*0.1)^2)/(18*α*α*0.1*0.1)),(((2+2*0.1-
α*0.1*0.1)^2-18*α*0.1*0.1*0.01)/(18*α*0.1*0.1)),((1+(-2+4*α*0.1-
4*α*α*0.1*0.1)*0.01)/(2-4*α*0.1+4*α*α*0.1*0.1)),(((-3+(-
1+α)*0.1)^2)/(18*(1-(1+α)*0.1+2*α*0.1*0.1))-(2*0.01))},{α,0,1}]

Plot[{(((2+2*α*0.2-α*α*0.2*0.2)^2)/(18*α*α*0.2*0.2)),(((2+2*0.2-
α*0.2*0.2)^2-18*α*0.2*0.2*0.01)/(18*α*0.2*0.2)),((1+(-2+4*α*0.2-
4*α*α*0.2*0.2)*0.01)/(2-4*α*0.2+4*α*α*0.2*0.2)),(((-3+(-
1+α)*0.2)^2)/(18*(1-(1+α)*0.2+2*α*0.2*0.2))-(2*0.01))},{α,0,1}]

Plot[{(((2+2*α*0.3-α*α*0.3*0.3)^2)/(18*α*α*0.3*0.3)),(((2+2*0.3-
α*0.3*0.3)^2-18*α*0.3*0.3*0.01)/(18*α*0.3*0.3)),((1+(-2+4*α*0.3-
4*α*α*0.3*0.3)*0.01)/(2-4*α*0.3+4*α*α*0.3*0.3)),(((-3+(-
1+α)*0.3)^2)/(18*(1-(1+α)*0.3+2*α*0.3*0.3))-(2*0.01))},{α,0,1}]

Plot[{(((2+2*α*0.4-α*α*0.4*0.4)^2)/(18*α*α*0.4*0.4)),(((2+2*0.4-
α*0.4*0.4)^2-18*α*0.4*0.4*0.01)/(18*α*0.4*0.4)),((1+(-2+4*α*0.4-
4*α*α*0.4*0.4)*0.01)/(2-4*α*0.4+4*α*α*0.4*0.4)),(((-3+(-
1+α)*0.4)^2)/(18*(1-(1+α)*0.4+2*α*0.4*0.4))-(2*0.01))},{α,0,1}]

Plot[{(((2+2*α*0.6-α*α*0.6*0.6)^2)/(18*α*α*0.6*0.6)),(((2+2*0.6-
α*0.6*0.6)^2-18*α*0.6*0.6*0.01)/(18*α*0.6*0.6)),((1+(-2+4*α*0.6-
4*α*α*0.6*0.6)*0.01)/(2-4*α*0.6+4*α*α*0.6*0.6)),(((-3+(-
1+α)*0.6)^2)/(18*(1-(1+α)*0.6+2*α*0.6*0.6))-(2*0.01))},{α,0,1}]

Plot[{(((2+2*α*0.7-α*α*0.7*0.7)^2)/(18*α*α*0.7*0.7)),(((2+2*0.7-
α*0.7*0.7)^2-18*α*0.7*0.7*0.01)/(18*α*0.7*0.7)),((1+(-2+4*α*0.7-
4*α*α*0.7*0.7)*0.01)/(2-4*α*0.7+4*α*α*0.7*0.7)),(((-3+(-
1+α)*0.7)^2)/(18*(1-(1+α)*0.7+2*α*0.7*0.7))-(2*0.01))},{α,0,1}]

Plot[{(((2+2*α*0.8-α*α*0.8*0.8)^2)/(18*α*α*0.8*0.8)),(((2+2*0.8-
α*0.8*0.8)^2-18*α*0.8*0.8*0.01)/(18*α*0.8*0.8)),((1+(-2+4*α*0.8-
4*α*α*0.8*0.8)*0.01)/(2-4*α*0.8+4*α*α*0.8*0.8)),(((-3+(-
1+α)*0.8)^2)/(18*(1-(1+α)*0.8+2*α*0.8*0.8))-(2*0.01))},{α,0,1}]

Plot[{(((2+2*α*0.9-α*α*0.9*0.9)^2)/(18*α*α*0.9*0.9)),(((2+2*0.9-
α*0.9*0.9)^2-18*α*0.9*0.9*0.01)/(18*α*0.9*0.9)),((1+(-2+4*α*0.9-
4*α*α*0.9*0.9)*0.01)/(2-4*α*0.9+4*α*α*0.9*0.9)),(((-3+(-
1+α)*0.9)^2)/(18*(1-(1+α)*0.9+2*α*0.9*0.9))-(2*0.01))},{α,0,1}]

Plot[{(((2+2*α*0.98-α*α*0.98*0.98)^2)/(18*α*α*0.98*0.98)),(((2+2*0.98-
α*0.98*0.98)^2-18*α*0.98*0.98*0.01)/(18*α*0.98*0.98)),((1+(-2+4*α*0.98-
4*α*α*0.98*0.98)*0.01)/(2-4*α*0.98+4*α*α*0.98*0.98)),(((-3+(-
1+α)*0.98)^2)/(18*(1-(1+α)*0.98+2*α*0.98*0.98))-(2*0.01))},{α,0,1}]
```

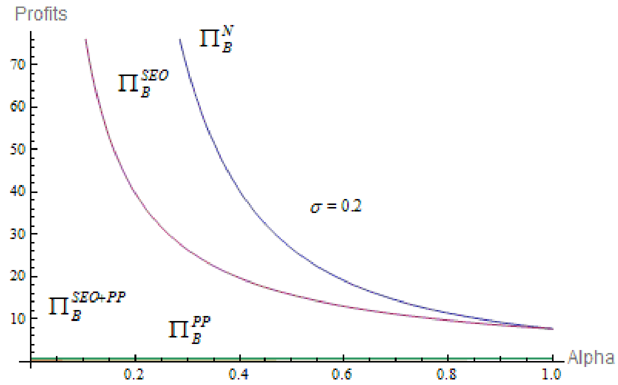
Simulations of the profit functions of advertisers A and B depending on the value of σ

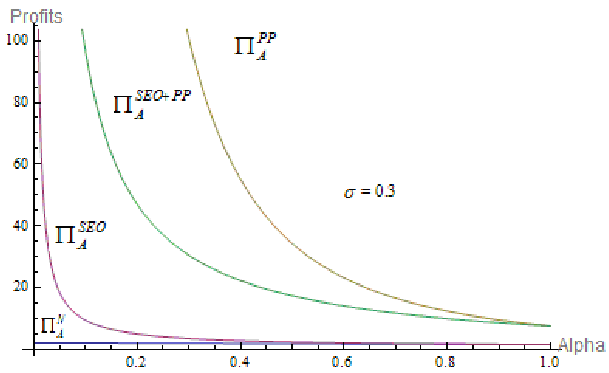
Given advertiser B doesn't invest in any SEM, the profit functions for advertiser A is	*Given advertiser A invests in PP, the profit functions for advertiser B is*

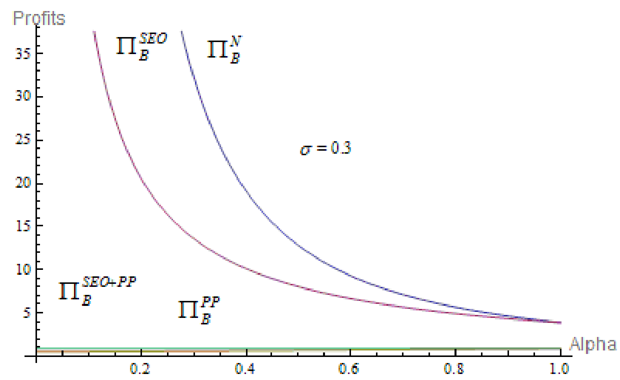


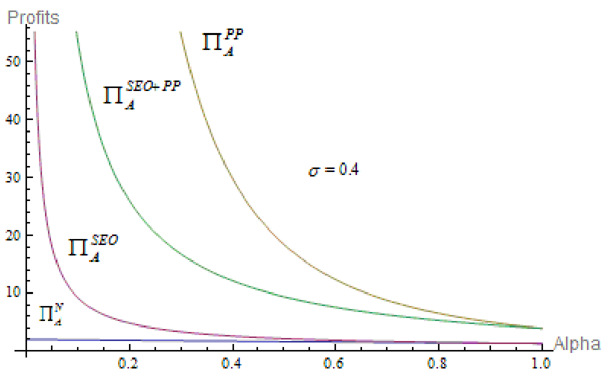


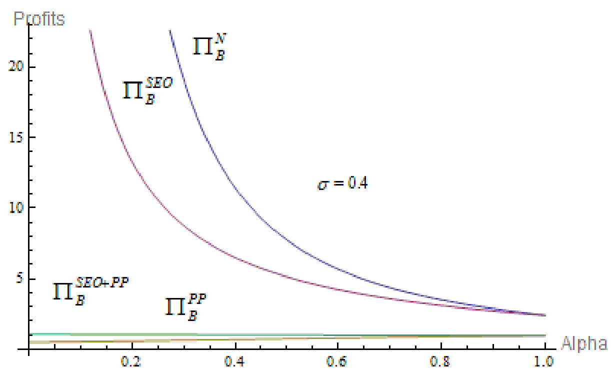


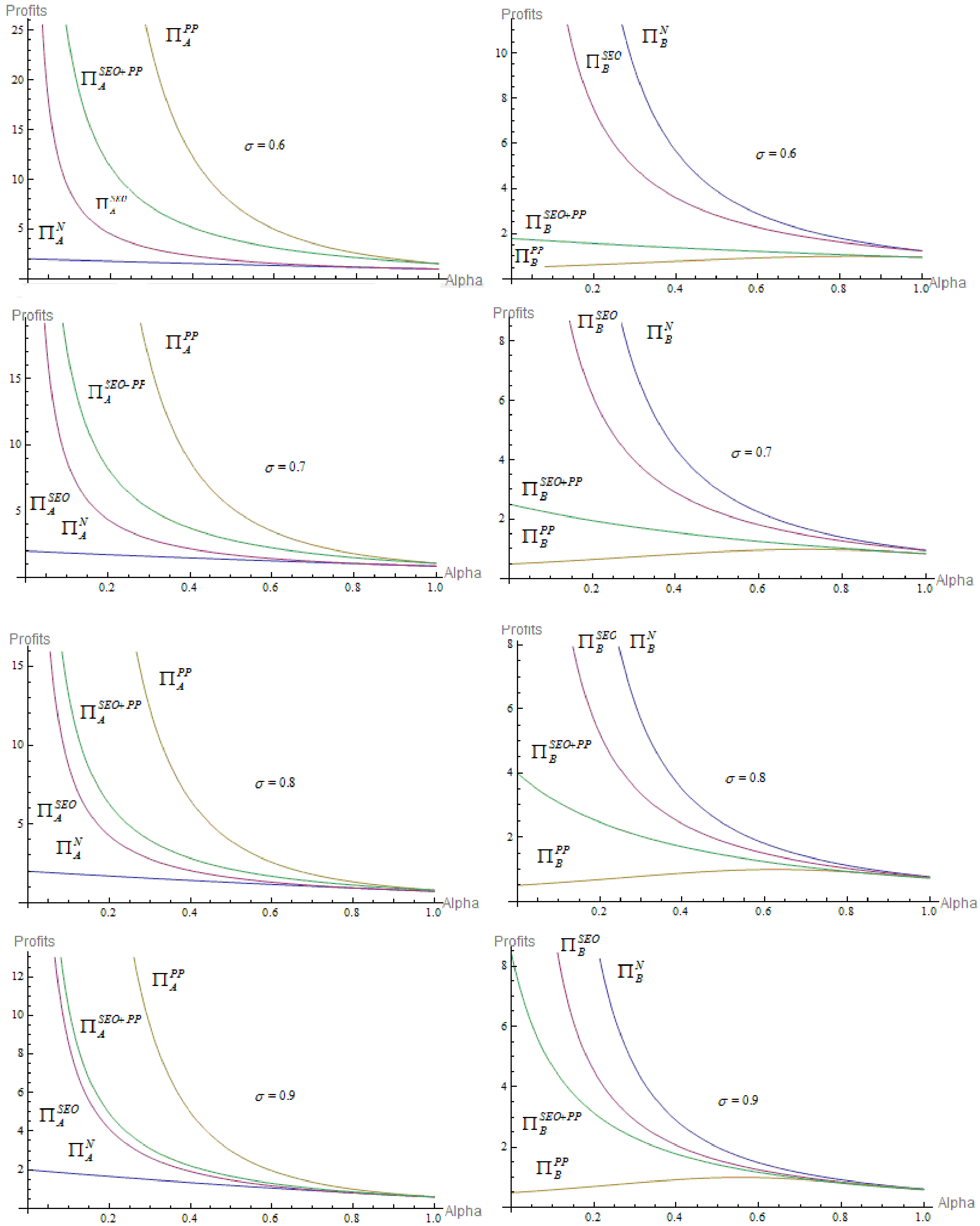


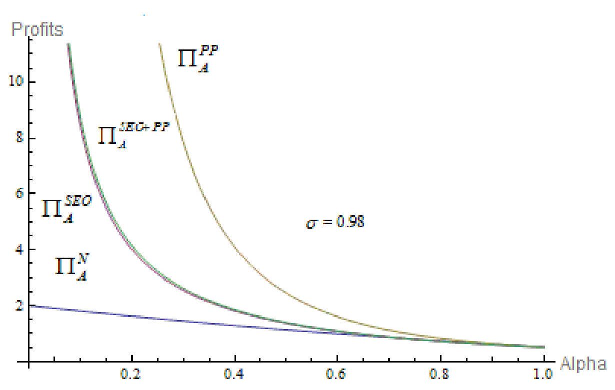


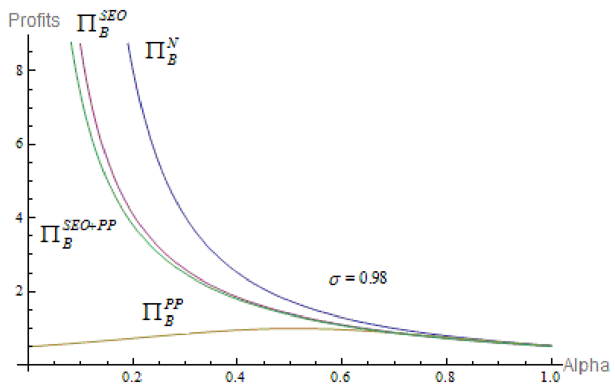







Profits
Alpha
Π_A^{PP}
Π_A^{SEO+PP}
Π_A^{SEO}
Π_A^{N}
$\sigma = 0.6$
Π_B^{N}
Π_B^{SEO}
Π_B^{SEO+PP}
Π_B^{PP}
$\sigma = 0.7$
$\sigma = 0.8$
$\sigma = 0.9$





Appendix C

Advertisers' Payoff Matrix

Advertiser B	Advertiser A			
SEM strategy	Do nothing	SEO	PP	SEO+PP
Do nothing	1.90 1.90	9.10 2.81	338.42 97.77	34.21 9.57
SEO	2.81 9.10	1.12 1.12	20.32 19.66	2.33 1.67
PP	97.77 338.42	19.66 20.32	0.54 0.54	1.30 1.31
SEO+PP	9.57 34.21	1.67 2.33	1.31 1.30	0.98 0.98

Notes: $\alpha = 0.1, \sigma = 0.5, c_i = 0.01$.

Advertisers' Payoff Matrix

Advertiser B	Advertiser A			
SEM strategy	Do nothing	SEO	PP	SEO+PP
Do nothing	1.82 1.82	8.57 2.52	100.50 32.35	10.42 3.02
SEO	2.52 8.57	0.60 0.60	3.56 9.48	0.60 0.60
PP	32.35 100.50	9.48 3.56	0.59 0.59	4.67 4.68
SEO+PP	3.02 10.42	0.60 0.60	4.68 4.67	0.59 0.59

Notes: $\alpha = 0.1, \sigma = 0.9, c_i = 0.01$.

Advertisers' Payoff Matrix

Advertiser B	Advertiser A			
SEM strategy	Do nothing	SEO	PP	SEO+PP
Do nothing	1.60 1.60	1.80 1.38	16.54 7.11	11.68 4.90
SEO	1.38 1.80	1.44 1.44	10.57 5.67	7.50 3.88
PP	7.11 16.54	5.67 10.57	0.74 0.74	0.84 0.85
SEO+PP	4.90 11.68	3.88 7.50	0.85 0.84	0.84 0.84

Notes: $\alpha = 0.7, \sigma = 0.3, c_i = 0.01$.

Advertisers' Payoff Matrix

Advertiser B	Advertiser A			
SEM strategy	Do nothing	SEO	PP	SEO+PP
Do nothing	1.55 1.55	2.30 1.15	12.33 5.66	5.16 2.11
SEO	1.15 2.30	0.97 0.97	3.33 3.59	1.52 1.23
PP	5.66 12.33	3.59 3.33	0.78 0.78	1.38 1.39
SEO+PP	2.11 5.16	1.23 1.52	1.39 1.38	0.94 0.94

Notes: $\alpha = 0.4, \sigma = 0.6, c_i = 0.01$.

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7 Glossary

302 Redirect – is the process when a server sends to a browser the location of a requested ad, instead of sending the ad itself. Ad servers use 302 redirects to allow tracking activities, such as ad requests or ad clicks.

Ad Impression – is an ad that is served to a user's browser.

AdSense - the Google Ad Sense program delivers Google Ad words that are added on the individual websites, and then Google pays to the web publishers for the ads, displayed on their sites, based on users' clicks on ads or ads impressions, depending on the type of ads. It allows websites' owners to display their advertisements on their web sites and get payments each time ads are clicked.

AdWords - The Google Ad Words program allows advertisers to display their advertisements in the Google content network through cost per click scheme.

Algorithm robustness - is a component of the Internet search engine quality from the manufacture's perspective in Garvin's (1984) framework. Higher algorithm robustness implies a greater conformance to the search engine's ranking specification and lower vulnerability to "noises", while lower algorithm robustness renders the information "manufactured" less predictable, more contaminated by "noises".

Black hat SEO - is the opposite of *White hat SEO*. Black Hat SEO can be any optimization tactic that gives a site a possibility to get the higher ranking position than it would get according to the quality of its content; doesn't improve the relevance of the site. In other words, Black hat SEO is an optimization tool, the usage of which is forbidden by the search engine's guidelines. If the site that uses Black hat SEO is caught by the SE, it can be penalized or even be removed from the index. 302 Redirect, buying external links (back links) are some black hat SEO techniques.

Click through Rate – is the response rate of an online advertisement, expressed as a percentage ratio, and calculated by dividing the number of clickthroughs, the ad received, by the number of impressions, multiplying by 100 to get a percentage.

Click through Ratio (CTR) - is the ratio of ads clicked on the total ads displayed. The click-through ratio is one of the measures of the effectiveness of advertising.

Cost per Click (CPC) – is an advertising fee, associated mostly with the keyword campaigns on search engines like Google and Yahoo!. Advertisers pay a search engine or an ad network for each click they get. Prices are typically auction-based.

Crawling algorithm - algorithm called On-line Page Importance Computation (OPIC). In OPIC each web page is given an initial sum of 'cash' that is distributed equally among the pages it points to (see *Web crawling*).

External links (back links) - are links to the given website. The search engine optimization can affect the number and quality of backlinks, which a site has, since some search engines provide significant weight to the backlinks of a site (in different cases can be both black hat SEO and white hat SEO techniques).

Generalized second price auction (GSP) - is an auction mechanism, where each bidder places a bid. The highest bidder obtains the first slot, the second-highest bidder obtains the second slot and so on, but the highest bidder pays the price bid of the second-highest bidder, the second-highest pays the price bid of the third-highest, and so on. GSP is used mainly in the context of keyword auctions, where sponsored search slots are sold on the auction basis (Google's AdWords technology).

Google Penguin - is a code name for a Google algorithm update that was first released on April 24, 2012. The update is aimed at decreasing organic ranking positions of those websites, which use declared black-hat SEO techniques, such as keyword stuffing, deliberate creation of duplicate content, cloaking, participating in link schemes, etc.

Keyword – is specific word entered into a search engine by the user that results in a list of websites related to this key word. Keywords can be purchased by advertisers in order to insert ads, which link to the advertiser's site within search results.

Meta Tags - is a special element of HTML that describes the content of the web page. It is placed at the beginning of the web page's source code. Meta tags belong to the White hat SEO technique and are very important for search engine optimization, and Meta tags facilitate the search engines' index of the pages by subject.

Panda algorithm - is a change to Google's search results ranking algorithm, which was first announced in February 2011. The change aimed to lower the rank of 'low quality websites', and place 'higher quality websites' in the top of the search results.

Pay-Per-Click – is an advertising pricing model, in which advertisers pay agencies or media companies for each click on an online ad or e-mail message.

Ranking algorithm – it is an algorithm that selects and put in the top positions the pages, which can more likely satisfy the user's needs.

Reserve price –is a hidden minimum price and the lowest price that the seller can accept for the good in the auction. If the listing ends without any bids that meet the reserve price, the seller can not sell the good.

Search engine (SE) - is an online tool that provides users with information, they search via the Internet or within a specific website. Normally, the user types a word or phrase, also called a search query, into a search box, and the search engine displays links to relevant web pages or site content.

Search Engine Optimization (SEO) –makes changes on the individual web pages in order to improve their positioning within one or more search engines.

Search Query - is a term, which is typed into a search text box. The query can be a one word, part of a word, phrase etc.

SEO copywriting – is a technique, which implies writing the content for the website, well readable for a page viewer, that contains selected keywords and phrases that the site's owner wants to use to increase the sites rank in online search.

Traffic – is the volume of visitors who search on the website. Traffic is the currency of online success, but it is not the only one indicator of it.

Web crawling or spidering - are mainly used to create a copy of all visited pages in order the search engine could process indexing the downloaded pages to provide faster online searches.

White hat SEO - refers to the usage of SEO strategies, techniques and tactics that focus on a human audience and completely follows the search engine’s guideline with the rules and policies (SEO copywriting, Meta tags are example of white hat SEO).