# Evaluating Catastrophic Risk and CAT Bonds Pricing Methods

*By*

**Jaurès NGOUFFO ZANGUE**

Master Degree in Economics, Econometrics & Financial Markets,

Ca'Foscari University of Venice.

June 2016

*Advisors:*

**Pr. Roberto CASARIN**

Ca'Foscari University of Venice, IMEF, & GRETA Association.

**Dr. Martina Nardon**

Ca'Foscari University of Venice, Department of Economics.

Thesis submitted in fulfillment of the requirement for the Master degree in Economics, Econometrics and Finance of Ca'Foscari University of Venice (Italy), joint program with the Master II in Econometrics and Financial markets of Aix-Marseille School of Economics (France).

# Acknowledgements

I would like to thank my supervisors Pr. Roberto Casarin and Dr Martina Nardon, for giving me an interesting topic. It was great challenge for me work on a topic related to actuarial science which was a new for me. You have always been there when I needed help, or when I just wanted a little discussion.

I am also very grateful for my wonderful program directors Pr. Eric Girardin and Pr. Monica Billio for giving the chance and opportunity to participate to this wonderful and rewarding exchange program between the master degree in Econometrics of Banking and Financial markets of Aix-Marseille School of Economics (France), and the master degree in Economics, Econometrics, and Finance of Ca'Foscari University of Venice (Italy).

A big thank you to Arnault, Jeannot, Cedric, Loc, and Komla. Your support and friendship have been of big impact. You have always been there for me, making sure that I got through. I also thank all my friend I did not cite their name, those who believed in me and supported and have always been there for me.

I am also extremely grateful for the support and interest my family have shown. Thank you to my brothers Plentis and Leos for always encouraging me. Thank you to my mother for telling me to use my head and always work hard. I also thank you Nirmala, for all your support.

# Abstract

Many studies and reports have shown that the number of natural disasters such as hurricanes, floods, tsunamis, droughts has increased this past years and caused a significant number of property damages and victims. The Hurricane Andrew experience has shown that the normal reinsurance system was too fragile to absorb the huge amount of losses of a natural disaster. The study of catastrophe risk is of major importance, if we want to prevent and mitigate the effects of a natural disaster both for the insurers and the insureds. The main purpose of this work is to find a proper way to evaluate catastrophic risk and to price CAT bonds. To this aim we will review the instruments used to hedge catastrophe risk such as CAT bonds. We will next review the state of art of pricing CAT bonds and focused on the probability distortion approach. Finally, we will estimate the risk adjusted parameter and the degree of freedom of the Student-t distribution by calibrating the the 2-factor Wang transform model.

**Keywords:** Catastrophe risk, CAT bonds pricing, Wang transform, premium, risk-adjusted parameter.

# Contents

# List of Figures

# Chapter 1

# introduction

This past decades, the insurance sector has been characterized by a boom of his activities, with the development of new insurance products. The wealth created by the overall growth of economies, the constant increase of the world population expose insurers to an increasingly important risk of insolvency in case of occurrence of a disaster. The expansion of human activity in our days has adverse effects on the environment resulting in global warming. With climate change, people and goods are increasingly exposed to the risk of a natural disaster. Due to the development of the insurance industry in developed countries, people and governments can easily hedge probable losses and damages from catastrophic event of a certain intensity by purchasing an insurance contracts and by that transferring their catastrophe risk to the insurers. To deal with catastrophic risk, insurers must quantify the risk they carry in order to be able to better manage it. The approach in the insurance world is to always give priority to the risk management because the slightest negligence may have disastrous and irreversible effects. A natural disaster can be defined as an event due to uncontrollable and destructive natural phenomena. Property Claim Services defines a catastrophe as an event that causes $ 25 millions or more in insured property losses and affects a significance numbers of property, casualty policyholders and insurers. Based on the amount , the recent flood in Paris in June 2016 which causes losses for about 1 billion euro can be considered as a natural catastrophe. The management of catastrophe risk implies taking into account its dimensional character. Normal insurance seems to successfully work for high frequency, low severity, relatively stationary independent event with good data, limited loss volatilities and identical probability distribution Cummins *et al.* (2006). For the case of catastrophic event, we are completely in an opposite situation. Catastrophe risks are extreme risk; that means they are of low loss frequency and high loss severity with unlimited loss. In this case, the traditional solutions of hedging and risk transfer by purchasing reinsurance contracts have been proved to be ineffective. Such practices could be less cost-effective to the reinsurance company, but can pose a severe financial stress to the reinsurance company due to the unpredictable nature of large catastrophic loss.

The main purpose of insurance company is to find a better way of hedging and transferring this types of extreme risk. Seeking for new funding and avoid being insolvent, insurance companies launch new products call Insurance-Linked-Securities (ILS). They are traded securities on the financial markets through securitization. The losses caused by catastrophic events could also lead to a significant amount of payment for the capital market investors. One of the most important ILS are catastrophe risk bonds commonly called CAT bonds. CAT bonds were developed to ease the transfer of catastrophic insurance risk from insurers and corporation to capital market investors. They protect corporation or insurance companies from financial losses caused by natural disasters, by offering an alternative or complement of capital to the traditional reinsurance. As the occurrence of catastrophe is largely unpredictable, valuing CAT Bonds is very difficult. The main aim of this work is to present an evaluation framework for the catastrophe risk and determine a proper model for the premium computation on the CAT bonds market. In order to realize the objectives of this work, we will do a state of the art on CAT bonds, and describe the process of issuance of CAT bonds. The second part will be devoted to the modelling of the catastrophe risk and pricing the CAT bonds.

# Chapter 2

# Catastrophe derivatives

In the last fifty years, the world's wealth increased significantly as well as its population[1]. To date, the world is facing a very important problem of climate change, and considering that cities with strong economic activity are generally density populated areas in case of occurrence of a natural disaster, human and material losses can be very important. Therefore insurance companies are exposed to the risk of insolvency because of the astronomical nature of losses resulting from a natural disaster. In order to deal with this risk, insurers must quantify the risk they carry to better manage it. Some catastrophe derivatives have been developed to help insurers to hedge from catastrophe risk. Catastrophe derivatives are capital market instruments that allow investors, which are not part of the insurance or reinsurance market, to invest in natural catastrophe risks. In order to study the catastrophe derivatives, we need to well define and understand the concept of catastrophe risk.

## 2.1 Catastrophe risk

A catastrophe risk is the risk associated with the losses in case of some natural disaster. In most cases, a natural disaster usually causes important property damages, economic damage or loss of life. The most common natural disasters which usually occur with great damage are: hurricanes, earthquakes, tsunamis, wildfires, droughts, avalanches,

[1]According to the World Bank the world population has passed from 2.52 billions in 1950 to 7.24 billions in 2015.

extreme temperature, tornadoes and floods. A study of the National Oceanic and Atmospheric Administration (NOAA) shows that this past decade, more people moved to coastal area like the Atlantic Coast and Gulf of Mexico, where hurricane threats are the greatest (Ou Yang 2010). A 2014 United Nations report on world urbanization gives the top thirty most populated cities in the world that year and Tokyo is at the top of the list. Japan is considered as a country where earthquake threats are very high. The increase of the population in this area, where there is significant risk of natural disaster has a substantial effect on the amount of claims and property damages in case of natural disaster. Since these highly populated areas are generally affected by natural disaster, people purchase insurance contracts to hedge the catastrophe risk. The increase of insured people will therefore increase the insurers catastrophe risk. Therefore, to model the number of claims we need to take into the correlation amongst the claims and the severity of the events. Some studies usually assume that claims are independent which is not completely right since the claims are the result of an catastrophic event which affects an entire region. We can also talk about a spacial correlation. The severity stand for the amount of claims. It is a key variable because losses from a natural disaster or man-made catastrophe are usually very large and even unlimited. If we consider cases of explosion of an oil rig, the damages on the ecosystem and the coastal population are usually extremely important and irreversible as regards to the effects on ecosystem. The Chernobyl disaster which is a man-made catastrophe causes huge losses of farmland, destruction of the ecosystem and important loss of life. To date, the expenses to cover the damage of the Chernobyl catastrophe are still taking place. Catastrophe risk will continue to grow as people will continue to migrate towards high risk area and as long as the effects on human activities on climate change will not reduce. The 2015 report of Swiss Re on natural Catastrophes shows that this past year, the number of natural disaster has increased and so the insurers amount of losses (see Figure 2.1).

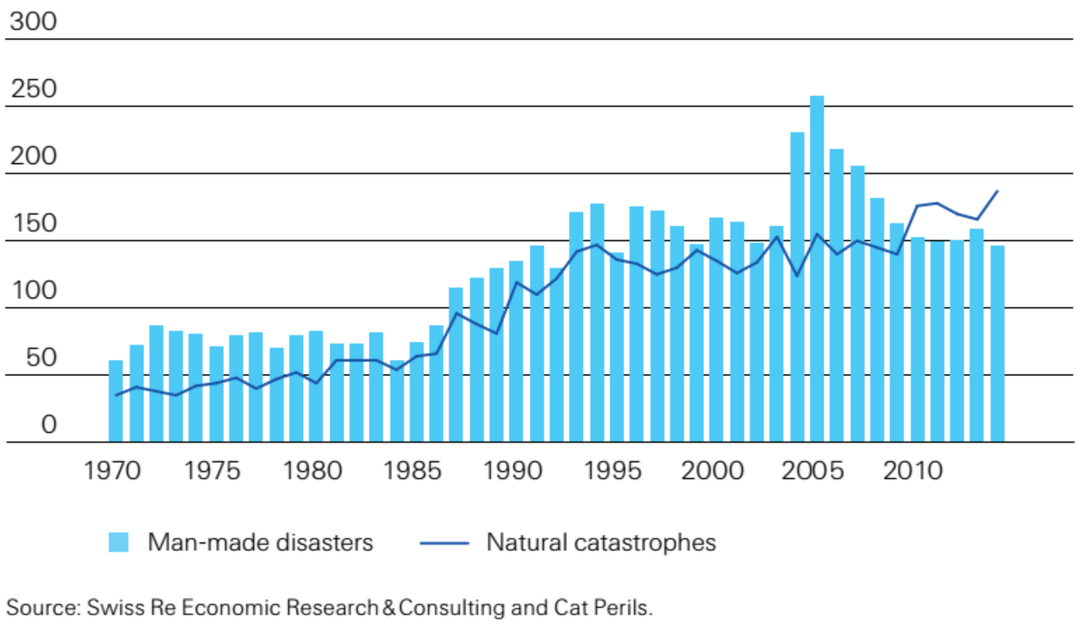



Figure 2.1: Number of catastrophe events world wide between 1970 - 2014

Swiss Re think the high occurrence of natural disaster can be contributed to the fact that cities are becoming larger, more populated, which causes a greater effect on climate changes. As climate changes, natural catastrophes are a significant issue for both developed and developing countries. Swiss Re also shows that, despite of the fact that insurers face very huge amount of losses after a natural disaster, the uninsured losses are by far more important than the insured ones. For example, the Japan earthquake tsunami of 2011 considered as the most costly natural disaster of this past thirty years with an overall losses estimated at US$ 210 billions[2] and the insured losses representing only US$ 40 billions. Hurricane Katrina overall losses was US$ 125 billions with an insured part of US$ 60.5 billions. Figure 2.3 gives a graphic representation of the world wide insured and uninsured catastrophe losses between 1970 and 2014.

Natural catastrophes are extreme events, that means events with low frequency and very high severity in terms of economic consequences. Swiss Re last report on natural catastrophe shows that this past decade the severity and frequency of natural disasters are increasing. Insurers are therefore constantly exposed to catastrophe risk. To date many tools have been developed to better understand the nature and consequences of natural hazard damage. In developed countries many instruments have been created

[2]Source: Munich Re, NatCAtService,2016.

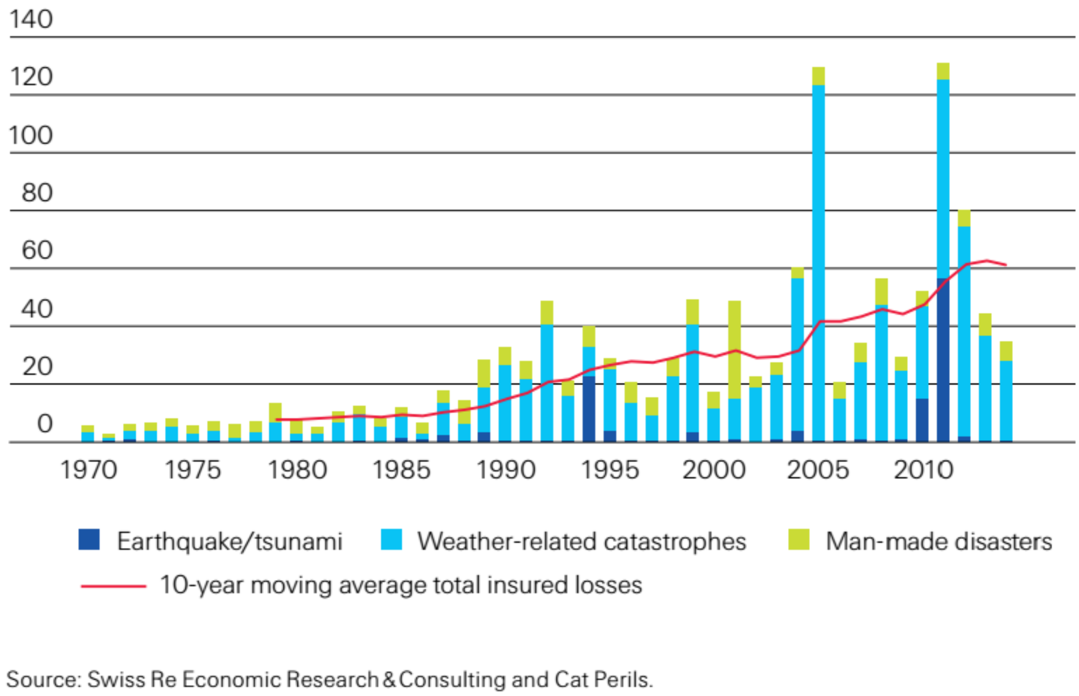



Figure 2.2: Insured world catastrophe losses between 1970 - 2014 in USD billion at 2014 price

to facilitate reconstruction after natural disaster, but it is not the case for developing country. This instruments are used by insurers for hedging and transferring catastrophe risk. As example of hedging and risk transfer instruments we can cite reinsurance and Insurance Linked Securities. The development of insurance sector in a country can be considered as an indicator of development.

Reinsurance is an insurance contract purchased by an insurance company called the cedent from another insurance company (the reinsurer) with the means of hedging and transferring a part, or hold the risk that the cedent cannot easily bear. Reinsurance contracts are less cost-effective for reinsurance company, but for risk like catastrophe risk, the reinsurer can face severe financial stress due to the unpredictable nature of natural disaster losses. The incapacity of the normal reinsurance market to hedge catastrophic risk was proved in the past, with the large amount of loss registered by the insurance industry after the Hurricane Andrew in 1992 in Florida (USA). Eleven American insurance companies went bankrupt. The reinsurance system was not able to handle the losses. Many insurance companies decided to abandon the hedging of catastrophe risk by reinsurance. The increasing need of a new source of hedging from catastrophe risks allowed to develop Insurance-Linked Securities. Insurance-Linked-Securities (ILS) are securities linked to an insurance risk issued on the capital market. There are financial instruments created by the process of securitization[3]. ILS are more sustainable to deal with catastrophe risk. The remainder of this section will be mainly devoted to the most popular ILS, the catastrophe bonds (CAT bonds) and others ILS and CAT options.

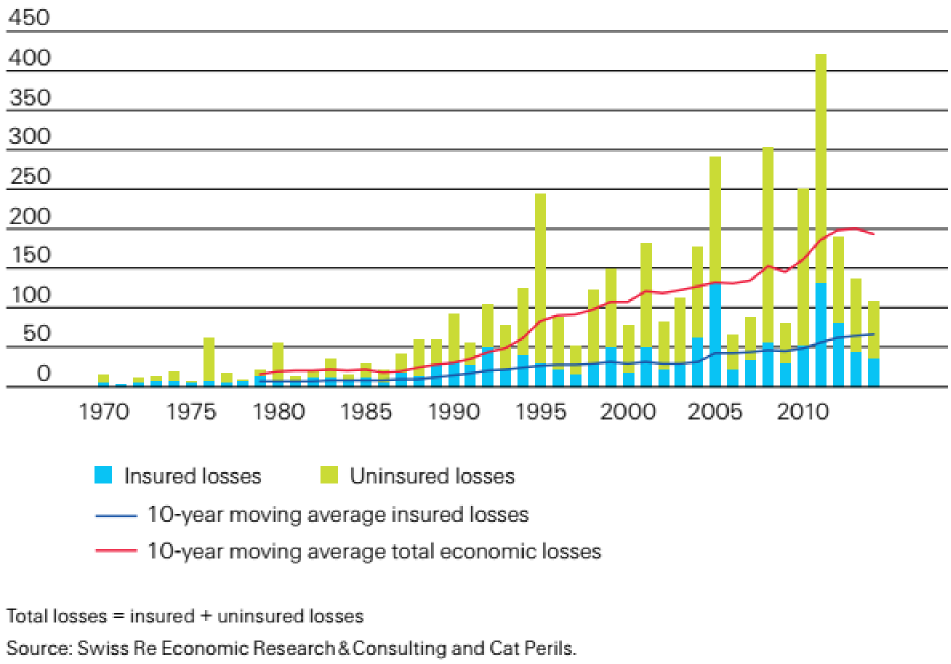



Figure 2.3: Insured and uninsured world catastrophe losses between 1970 - 2014 in USD billion at 2015 price

## 2.2 CAT bond principles

Catastrophe bonds are the most popular and well understood Insurance-linked securities for managing CAT risk Garg (2008). Their purpose is to crowd-source reinsurance coverage, in order to reduce reinsurers, insurers, and self-insurers reserve requirements and reduce their cost of coverage. CAT bonds are risk-linked securities used by insurers, reinsurers, governments, and corporations (sponsor) to transfer a specific set of risk (usually catastrophe risk) to the financial market investors. The risk is then borne by the investors. The new holders (investors) of the CAT bonds are linked to a particular

[3]Securitization is the process where illiquid or untradable financial instruments are converted to a form that allows for a greater liquidity.

catastrophe event or natural disaster (hurricane, flood, etc). CAT bonds protect the sponsor(s) from the financial losses caused by a natural disaster. CAT bonds are particular bonds because their coupons and payments depend not only on the occurrence a catastrophe event but also on the gravity or severity of the related risk.
The first CAT bonds were issued just after the failure of the reinsurance market to cope with the losses of the hurricane Andrew in 1992. Hannover Re initiated the catastrophe bond market[4] in 1994, with a first issuance of about US$ 85 million, it was a successful experience. The first CAT bond issued by a non-financial firm was in 1999 to cover the earthquake losses in Tokyo region for Oriental Land company (Division 2002).

Figure 2.4 presents the structure of a CAT bond. The transaction involves a sponsor (government agencies, corporations, insurers, reinsurers) which seeks to transfer the risk, a Special Purpose Vehicle (SPV), the collateral and the investors. The sponsor transfers its risk to the capital market by setting a SPV. The latter will conduct two actions simultaneously: it will issue CAT bonds to the investors, and will be a source of reinsurance for the sponsor. The proceeds from the bonds issued are invested in high quality short term securities (such as US Treasury money market fund) and deposited in a collateral account. The earning from the high quality securities are swapped at the London Interbank Offered Rate (LIBOR) with a high rate swap counter-party. Since the LIBOR is a floating rate, in order to be covered again the interest rate risk, the trustee enter in a swap contract with a swap counter party. The trustee will paid a fixed return to the the swap counterparty and in return he will receive form him a the LIBOR plus a swap spread. This system enables the trustee to be always able to pay the SPV investment income. The swap is very important, because it is used to cover the sponsors and the investors for interest rate risk and default risk (Ma & Ma 2013). The sponsor enters into a reinsurance contract (alternative reinsurance) with the SPV and pays him a premium in order to be covered up to a given limit amount specified in the contract. Before entering in the alternative insurance contract, the SPV has to prove to the sponsor that he has a capital equal or greater than the limit amount of the coverage. Therefore the proceeds form the CAT bonds issuance plus the amount of the premium to be paid for the alternative reinsurance, all compound at the default-free interest rate has to equal

[4] Hannover Re Overview on ILS, NatCAt Exposure of 23 October 2013.

to the limit amount of the coverage. When the sponsor is assured by it, the alternative reinsurance contract is signed.

From the sponsor perspectives, CAT bonds provides full collateralized losses as compared to reinsurance; they eliminate the concerns about credit risk. The investors pay a principal to obtain the CAT Bond and receive as return a regular periodic payments, generally quarterly or semi-annually. The interest or the coupons paid to the investors are from the premium[5] and the proceeds form the investment bonds received by the SPV from the collateral.

If the trigger event (covered event) does not occur during the life of the CAT bond, the investors will receive the principal plus the final coupon or a generous interest (a compensation for the catastrophe risk exposure). The coupons are generally paid quarterly, but they can also be semi-annual or annual depending on the contract. CAT bonds are remunerated at LIBOR plus a yield (spread or risk premium). The spread is a remuneration for the unpredictable property of natural disasters. So the total coupon rate(%) is equal to the LIBOR(%) plus de spread(%). The spread is the price of the risk. Lane (2000) shows that the spread over the LIBOR of the CAT bonds coupon is equal in rate to the premium of the alternative insurance in rate. The LIBOR here is a floating rate. However if the covered event occurs with the contract specified triggers during the risk exposure period, the SPV pays the sponsor (ceding company) according to the terms of the reinsurance contract. Depending on the contract terms, the investors can receive a part of the principal and interest or nothing. CAT bonds can be issued to cover one peril or multiple perils. Investors generally prefer single peril CAT bond because they want to be able to construct their own portfolio of risk, while sponsors prefer multiple perils CAT bonds because they enable them to reduce transaction costs.

Every CAT bond issued is linked to a specific payout-trigger. The definition of the payout-trigger event has a key role in implementing CAT bonds. Guy Carpenter (2004) present the variety of trigger mechanism used to determine when the losses of a natural disaster should be covered by the CAT bond. We have *indemnity trigger*, *index triggers* and *hybrid triggers*.

[5] Amount pay by the sponsor to the SPV.

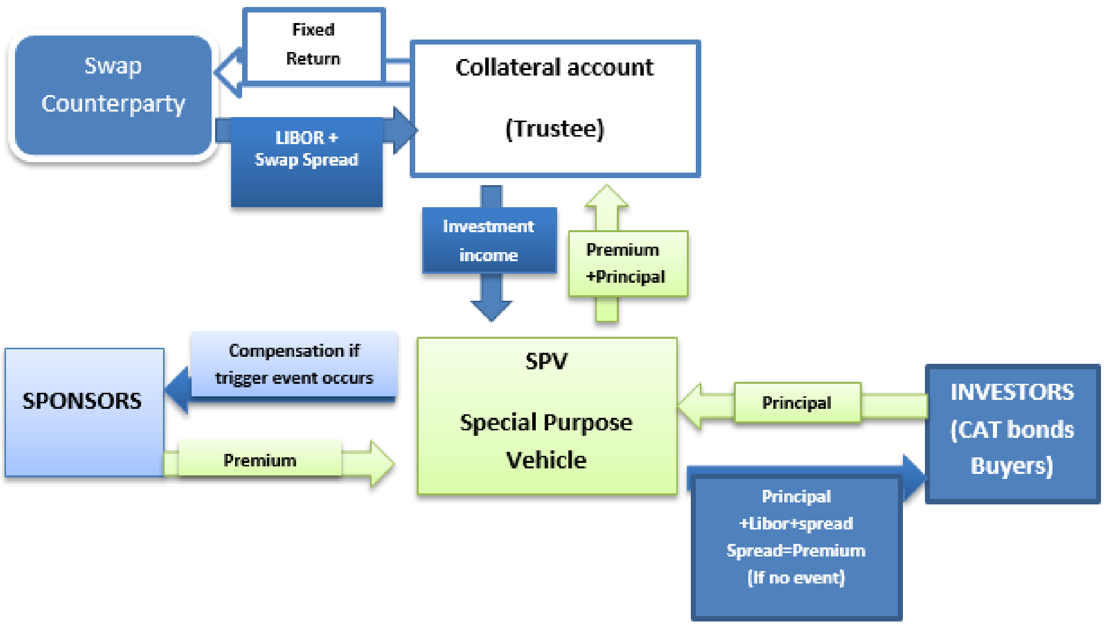



Figure 2.4: Structure of CAT bonds system

For *indemnity trigger*, the payouts are based on the sponsors actual losses. This type of trigger gives the sponsors the lowest possible level of basic risk[6], but it is also subject to a high level of moral hazard[7] problem. This is a reason why insurers and reinsurers often favour indemnity trigger (Cummins 2008). However this trigger requires the sponsor to disclose to the investor information on the risk exposure of their underwriting portfolio, and it may be very difficult for the sponsor. The indemnity trigger generally requires more time than the non-indemnity trigger to reach the final settlement. Investors prefer non-indemnity triggers or index trigger, because of their low moral hazard, high transparency, and better liquidity (McGhee *et al.* 2005). This type of trigger is very close to the traditional reinsurance protection. The *index triggers* exposes the sponsor to a high basis risk. They are composed of *industry loss index*, *parametric index*, and the *modeled-loss trigger*. In the case of industry loss trigger, the ceding company recovers a proportion of total industry losses in excess of a predetermined point to the extent of the remainder of the principal. For the parametric index trigger the bond payouts are triggered by a specified occurrence of a catastrophic event with a defined physical parameter and also

[6]Basis risk is the risk that, in the event of a covered loss, the payout determined by the bond calculation will differ from the actual loss incurred by the sponsor.

[7]It is a situation where the sponsor will no longer try to limit the potential losses since the risk is transferred to the investors.

takes into account the sponsors exposure to events in other areas. The specificity of this trigger is that the sponsor does not have to disclose confidential informations. In the case of the modelled-loss trigger, the trigger index is determined after the occurrence of the catastrophe. The physical parameters of the catastrophe are used to estimate the expected losses to the sponsor portfolio. The bond is triggered, if the modelled losses are above a specified threshold. Finally, the *hybrid trigger* is a particular trigger which is composed of more than one trigger for a single transaction.

The CAT bonds market is considered as an incomplete market, because the primary risk of CAT bonds is the occurrence of a catastrophe that triggers the loss principle. Since there are no securities other than CAT bonds whose payouts are contingent on the occurrence of a natural disaster, CAT bonds cannot be priced in terms of portfolio of the assets that are already traded and priced in the market (Cox & Pedersen 2000). CAT bonds and default bonds present some similarities. They are all high yield bonds. Defaultable bonds yield higher returns in part, because of their potential defaultability, while CAT bonds are offered high yield because of the stochastic nature of catastrophe process. CAT bonds are one of the most used catastrophe risk transfer derivatives. They present the advantage to be less correlated to stocks returns. The occurrnce of a natural disaster is not correlated with event in the board of the economy such as inflation, recession, interest rate movements, and stock market. From the CAPM[8] point of view, CAT bonds are zero-beta asset, a characteristic which makes then to be an excellent instrument for portfolio diversification. Froot (2001) thinks CAT events have a clear and direct effect on non-financial asset such as housing; so their correlation with financial asset can be misleading. The bankrupt of Lehman Brothers during the 2008 financial crisis, causes the default of the CAT bonds for which it was the trustee. During this period Lehman Brothers failed to honor its side of interest rate swaps. It was an exceptional case but it highlights the possible exposition of CAT bonds to systemic risk. Since this incidence with Lehman brothers, SPVs have taken measures to deposit the investor's principal in the safest security available. The market has changed the swap collateral and put it trust in other collateral solutions such as Treasury money market funds who is known as the most popular collateral solution, and triparty agreement.

[8]Capital Asset Pricing Model.

## 2.3 Others Insurance-linked Securities and Catastrophe options

In addition to CAT bonds, there are other insurance linked derivatives and instruments used to transfer catastrophe risk. In the following, we will briefly review some of this instruments.

### 2.3.1 Insurance-linked instruments

There are other insurance-linked derivatives and instruments used to transfer the catastrophe risk. In the following, we will review some of these instruments.

#### Industry Loss Warranties

Industry loss warranties (ILWs) are index-based reinsurance contracts. They cover the issuers from the occurrence of a catastrophe risk that may generate an industry loss of a pre-agreed size. ILW are duals triggers reinsurance contracts that have a retention trigger based on the incurred losses of the insurer buying the contract and also a warranty trigger based on the industry-wide loss index (Cummins 2008). Both triggers have to be hit in order for the insurer or the buyer of the contract to receive the pay off. They have a similar principle to CAT bonds industry index trigger. The industry loss trigger induces a reduction of the moral hazard problem but an increase of the basis risk. ILW are considered to be more flexible and easy to develop compared to the others forms of alternatives risk capital. Gatzert *et al.* (2007) gives a very good presentation of ILWs and the way they are used to hedge catastrophe risks. There are different types of ILWs[9]: *Life CAT Industry Warranty* contracts which are traded while the event is occurring, often while the occurrence of the event is certain; *Dead CAT Industry Loss Warranties* traded for an event which has already happened, but where the final loss are not yet known. *Back-up covers* are traded after the event has occurred to provide protection against follow-on events which certain catastrophe can cause.

[9]Source: www.artemis.com

### Sidecars

Sidecars are kind of reinsurance company such as hedge funds. They are created and funded by investors to provide capacity to single sponsor for hedging its catastrophic losses. Sidecars help insurer finance any type of risk in their books, including property risks. Sidecars play the role of the SPV in the case of CAT bonds. Bouriaux & MacMinn (2009) and Cummins (2008) give a detailed description of the functioning of Sidecars.

### 2.3.2 Catastrophe options

There are four main insurance-linked options uses for transferring catastrophe risk.

#### Event Loss Swaps

Event Loss Swaps (ELS) are CAT-linked derivatives launched by Deutsche Bank to help clients (insurers, reinsurers, corporations) to hedge against economic impact of US wind and earthquakes. ELS have a similar mechanism with credit default swaps. The buyer of the ELS contract has to pay a premium to the seller of the ELS, the latter will pay a notional value to the swap contract if the industry wide insurance faces losses due to a single catastrophe event that exceed a specified trigger level defined by a third party. Cummins (2008) describes a closed similar product to the ELS: the catastrophe risk swap. The catastrophe risk swap has the same principle with the Events Loss Swap.

#### NYMEX Risk Index Futures and Options

NYMEX[10] contracts are standardized futures and options contracts introduced for US hurricane risks. The indexes of industry losses are estimated by Aon Re from the Property Claims Services (PCS) data. NYMEX offers the futures contracts in the open-outcry and the options contracts on the GLOBEX electronic venue. The futures and options prices are based on market estimates of cumulative industry losses for catastrophes that occur during a calendar year. The contract settles in cash at the end of March of the following calendar year.

[10] New York Mercantile Exchange.

### CME Hurricane Futures and Options

The CME[11] Hurricane futures and options are designed in differently from those of NYMEX. CME products are one peril instruments, they settle against Carvill[12] Hurricane index, which is based on the parametric features of a hurricane. As soon as an official hurricane makes landfall, the CME futures and options expire. The contract settle in cash against the value of Carvill index, which is immediately released after the hurricane landfall.

### IFEX Event-Linked Futures

IFEX Event Linked Futures are triggered by insured losses as calculated by Property Claims Services (PCS). IFEX is a subsidiary of Climate Exchange PLC. The futures contracts are designed to mimic industry loss warranties with a payout linked to first event of the year , second even of the year and so on.The futures contracts settle against an industry wind loss estimated by PCS.

[11] Chicago Mercantile Exchange.

[12] Carvill Hurricane index is an index which describes the potential for damage from an Atlantic hurricane. Itis used as the basis for trading hurricane futures and options on the Chicago Mercantile Exchange (CME).

# Chapter 3

# Pricing Approaches

The pricing of catastrophe insurance-linked securities has a key role in the prevention and mitigation of catastrophe risk. Many academics and professionals have studied the properties of Insurance-linked securities and CAT bonds in particular; in order to develop proper methods to price their related risk. This chapter will be devoted to the state of the art of CAT bonds pricing. The pricing approach of CAT bonds depends on the context of the analysis. The literature does not gives a clear guidance for the valuation of CAT bonds. According to Cox & Pedersen (2000), the fact that the pricing of CAT bonds requires an incomplete market setting creates special difficulties in the pricing methodology. In the literature, we find the actuarial and the financial pricing approach. Within the actuarial approach a pricing method recently introduced is the one based on probability distortion operators to price CAT bonds. In the remainder of this chapter, we will first present the state of the art for the pricing methods with probability distortion function and next the actuarial and financial methods.

## 3.1 Pricing with Distortion Operators

Distortion operators have been used in insurance and finance to price risk. These methods apply distortion risk measures to price the risk of an insurance or a financial product. Wang (1995) present a framework for premium pricing principle in insurance, where the risk loading is imposed by a proportional decrease in hazard rates. Wang (1995) uses the proportional hazard (PH) transform to propose a risk-adjusted premium for pricing risk. With the PH transform the author shows that, for the same underlying risk, the risk-adjusted premium is larger for the party which is more risk adverse. The additive property of the PH transform makes the distortion operator to be very appealing to insurance risk. Wang (1996) discusses a class of premium principle using the PH transform. In this study the author presents the properties of the PH transform and its possible application depending on the distribution of the losses. He shows that the PH transform is comonotonic-additive and preserves the stochastic dominance, and reaches to the conclusion that it is a good method for pricing insurance risk.

In a period where financial risk and insurance risk were becoming more integrated, it is highly desirable to have a unified pricing theory (Wang 2000). The PH transform fails to replicate the Black-Scholes formula for pricing option, since the result of its application on a log-normal distributed series is no longer log-normal. Gerber *et al.* (1994) presents an application of the Esscher transform in a financial framework. The Esscher transform is distortion operator that takes a probability density function $f(x)$ and transform it into a new probability function $f(x, h)$ with a parameter $h$.

$$E_h[f(x)] = f(x,h) = \frac{e^{hx} f(x)}{\int_{-\infty}^{+\infty} e^{hx} f(x) dx} \tag{3.1}$$

Gerber *et al.* (1994) show that the Esscher tranform is an efficient technique for valuing derivative securities if the logarithm of the prices of the primitive securities are governed by certain stochastic processes with stationary and independent increments. This family of processes includes the Wiener process, the Poisson process, the gamma process, and the inverse Gaussian process. An Esscher transform of such a stock-price process induces an equivalent probability measure on the process. The Esscher parameter or parameter vector is determined so that the discounted price of each primitive security is a martingale under the new probability measure. The Esscher transform can reproduce the Black-Scholes option pricing formula. The effect of the Esscher transform on a normal distribution is shifting the mean.

$$E_h[N(\mu, \sigma^2)] = N(\mu + h\sigma^2, \sigma^2)$$

**Proof**

Let us assume a variable $X \sim N(\mu, \sigma^2)$. If we apply the Esscher transform on the probability density function of the variable $X$ we will have:

$$f(x,h) = \frac{e^{hx}\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\int_{-\infty}^{+\infty} e^{hx}\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx}$$

let us set the denominator of $f(x,h)$ equal to $M(h)$

$$M(h) = \int_{-\infty}^{+\infty} e^{hx}\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$$
$$M(h) = \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty} e^{hx}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$$
$$M(h) = \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty} e^{-(\frac{(x-\mu-\sigma^2h)^2-(\mu+\sigma^2h)^2)}{2\sigma^2})}dx$$
$$M(h) = e^{\mu^2h^2+\frac{\sigma^2h^2}{2}}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$$

we obtain:

$$f(x,h) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}e^{hx-\mu^2h^2-\frac{\sigma^2h^2}{2}}$$

$$f(x,h) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-(\mu+h\sigma^2))^2}{2\sigma^2}}$$

The function $f(x,h)$ is a normal density distribution of parameter $\mu^* = \mu + h\sigma^2$ and $\sigma^* = \sigma$.

Venter (1998) proposed to use the log-Esscher transform as an alternative to the PH-transform for lognormal risk. Nevertheless the Esscher transform presents some limitations which are related to the his derived risk measure called the Esscher principle. The Esscher principle does not respect the positive homogeneity property for $h > 0$. The premium cannot be computed under the Esscher principle when the claim follows a log-normal distribution, because the moment generating function of the log-normal does not exist. It is a serious drawback for the Esscher transform, since in the actual market both the financial and the insurance products often follows a log-normal distribution.

Wang (2000) presents the limitations of the previous distortion operators. He confirms the fact that since the PH transform fails to replicate the Black-Scholes formula for a log-normal risk, it cannot be applied simultaneously to assets and liabilities. Wang (2000) also presents a new class of distortion operator for pricing both insurance and financial risk called the Wang transform. He applies this new distortion operator to stock price distribution and recovers the risk neutral valuation for option and in particular the Black-Scholes formula. The Wang transform is presented as a sustainable approach combining both the actuarial pricing and financial pricing theory. Hamada & Sherris (2003) present a framework of pricing contingent claims using probability distortion operators. The authors used the Wang transform and extended it to a case where the underlying security risk has a time varying parameter. Wang (2000) presents a universal framework for pricing financial and insurance risk. He introduces a transfer and correlation measure that extend the CAPM to pricing of all kinds of assets and liabilities. He extends the CAPM to risk with non normal distribution and obtains a new parameter called the market price of risk. The parameter can be compared to the Sharpe ratio in case of normally distributed returns; it can be implied from, or implied to, a distribution in order to obtain a risk-adjusted price. Wang (2002a) applies the Wang transform to the pricing of call options on trading stocks and to pricing derivatives.

The Wang transform can be presented as follow. Let $g_\lambda(\cdot)$ be the Wang distortion operator defined as:

$$g_\lambda(v) = \Phi(\Phi^{-1}(v) + \lambda); \tag{3.2}$$

where $\Phi$ is a standard normal cumulative distribution function. Let us introduce the objective loss exceedance curve $S(x) = 1 - F(x)$ , where $F(x) = Pr(X < x)$ is the cumulative distribution function of a given loss variable $X$. There is no restriction as regarded to the type of distribution of $F(x)$. Wang (2000) presents the following universal pricing model based on the Wang transform:

$$S^*(x) = \Phi(\Phi^{-1}(S(x)) + \lambda). \tag{3.3}$$

If $X$ is an asset, the Wang transform will be:

$$F^*(x) = \Phi(\Phi^{-1}(F(x)) + \lambda); \tag{3.4}$$

or in terms of density function:

$$f^*(x) = \phi(\phi^{-1}(F(x)) + \lambda))\frac{1}{\phi(\phi^{-1}(F(x)))}f(x).$$

The mean value under $S^*(x)$ denoted by $E^*[X]$, will define a risk-adjusted fair value of X and $\lambda$ represents the market price of risk. It is worth noting that:

- if $F(x)$ has a normal$(\mu, \sigma^2)$, $F^*(x)$ is also a normal distribution with $\mu^* = \mu + \lambda\sigma$ and $\sigma^* = \sigma$;

- if $F(x)$ has a log-normal$(\mu, \sigma^2)$, such as $ln(X) \sim N(\mu, \sigma^2)$;$F^*(x)$ is also a log-normal of $\mu^* = \mu + \lambda\sigma$ and $\sigma^* = \sigma$.

For illustration, let us assume a given variable $X \sim N(\mu, \sigma^2)$; we have:

$$\begin{aligned} S_X(t) &= P(X \geq t) \\ &= 1 - P\Big(\frac{X-\mu}{\sigma} \leq \frac{t-\mu}{\sigma}\Big) \\ &= 1 - \Phi\Big(\frac{t-\mu}{\sigma}\Big). \end{aligned}$$

The Wang distorted decumulative distribution is :

$$\begin{aligned} g_\lambda(S_X(t)) &= \Phi[\Phi^{-1}(S_X(t)) + \lambda] \\ &= \Phi[\Phi^{-1}(1 - \Phi(\frac{t-\mu}{\sigma})) + \lambda] \\ &= \Phi[\Phi^{-1}(\Phi(\frac{-t+\mu}{\sigma})) + \lambda] \\ &= \Phi[-\frac{t-\mu-\sigma\lambda}{\sigma}] \\ &= 1 - \Phi[\frac{t-(\mu+\sigma\lambda)}{\sigma}] \\ &= S_Y(t), \end{aligned}$$

with $Y \sim N(\mu + \sigma\lambda, \sigma^2)$.

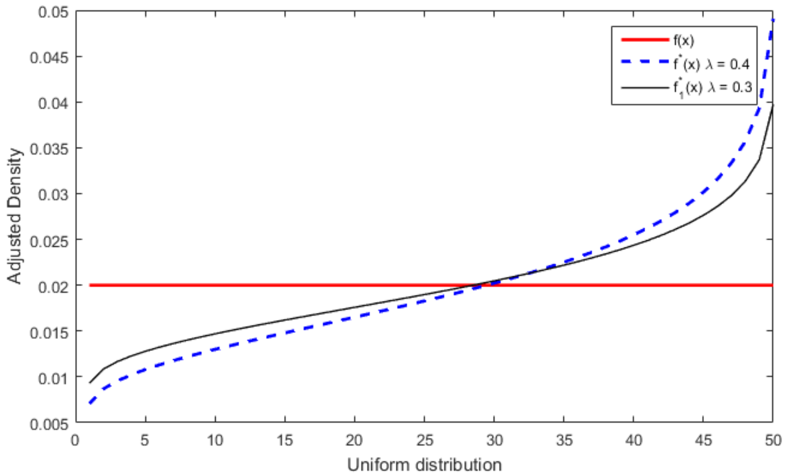



Figure 3.1: One factor Wang transform of a Uniform [1,50] distribution (displayed in terms of probability density)

Figure ??, shows that the one-factor Wang transform inflates probability density for adverses outcomes while deflating probability density for favorable outcomes, as a result it incoporates a form of risk loading or risk adjustment (Wang 2004). Following the above result, the Wang transform applied on the return $r_i$ of an asset $i$ will give:

$$E^*(r_i) = E(r_i) + \lambda\sigma_i. \tag{3.5}$$

Since the real moment of a the distribution of a given population are not generally observed, they are usually estimated using the sample observations. The probability assessment regarding the future outcome, highlights the importance of the Student-t distribution. In order to take in account the skewed property of the distribution distribution, Wang (2002a) suggests to replace the normal distribution by a t-Student distribution with $k$ degree of freedom. This transformation leads to the following two-factors Wang transform model:

$$S^*(x) = Q(\Phi^{-1}(S(x)) + \lambda), \tag{3.6}$$

where $Q$ has a Student-t distribution with k degree of freedom. Wang (2002a) reported that the two-factor model provides an excellent fit to the CAT-bond and corporate bond yield spread. Without the Student-t adjustment, the one-factor Wang transform (3.2) would not be able to explain the yield spreads in the CAT bonds and corporate bonds data. Using the historical default frequency of some corporate bond, Wang (2004) shows that the two-factor model provides a risk premium adjustment not only for the second moment but also for higher moment for parameter uncertainty [1].

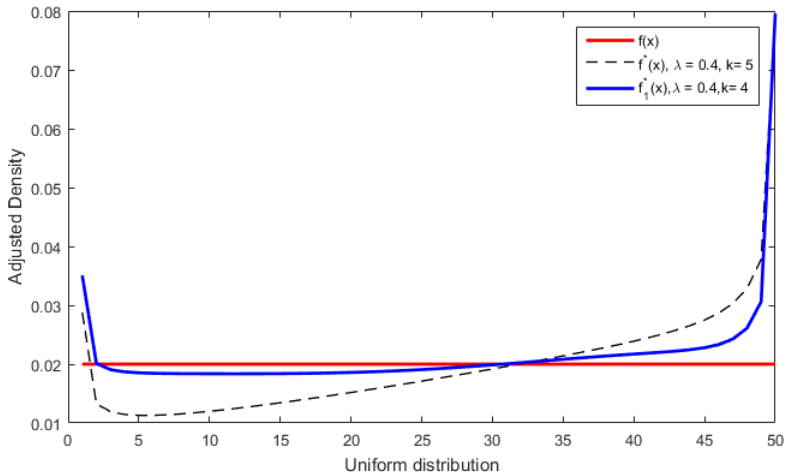



Figure 3.2: Two factors Wang transform of a Uniform [1,50] distribution (displayed in terms of probability density)

As shown in Figure 3.2, the 2-factor Wang transform takes in account the extreme tail of the probability distribution. In another words, this new transform inflates the probability density at both extreme tails, in order to take into account the so-called *greed and fear* investors' behavior. It is very consistent with volatility "smile"[2] in option prices. Wang (2004) uses this same method to compare CAT bonds and corporate bonds yield. He found that they both offer the same risk return trade-off in term of Sharpe ratio. Nevertheless, the difference in the degree of freedom of the Student-t distribution shows that the CAT bonds are more attractive to investors. Wang (2004) shows that the 2-factors Wang transform fits to the pricing of CAT bonds and corporate bonds. Wang transform seems to be a better way to actually price the implied yield spread over LIBOR of a CAT bond. However, the Wang transform has not been widely used in the literature for CAT bond pricing.

[1]In reality, the probability distributions are always estimated based on limited available data, so parameter uncertainty is always present. This is very recurrent in cat bond modelling.

[2]Volatility smiles are implied volatility patterns that arise in pricing financial options. In particular for a given expiration, options whose strike price differs substantially from the underlying asset's price command higher prices than what is suggested by standard option pricing models. These options are said to be either deep in-the-money or out-of-the-money.

Kijima (2006) extend the Esscher transform and Wang transform to a multivariate settings for pricing general financial and insurance risks. The author highlights the coincidence between the Esscher transform and Wang transform when the underlying risks are normally distributed. Kijima & Muromachi (2006) proposes a new probability distortion for the tail distribution to price financial and insurance risks. The new transformation derives from the Bühlman (1980) equilibrium pricing model. The authors present one of the drawbacks of the one factor Wang transform as the fact that his normality assumption never match with the fat tail distribution observed in financial markets. They then propose a new two-parameter transforms base on the idea of Kijima (2006) and obtain:

$$F^{**}(x) = P_{k;\delta}[Q_k^{-1}(F(x))], \tag{3.7}$$

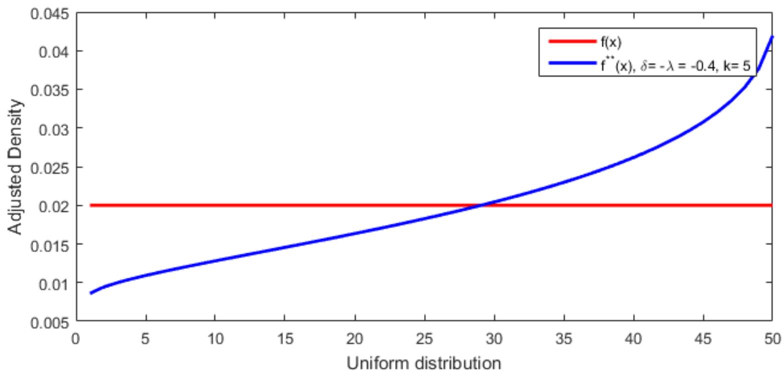



Figure 3.3: Extension of the Wang transform by Kijima of a Uniform [1,50] distribution (displayed in terms of probability density)

With $P_{k;\delta}$ representing a non central t-distribution with $k$ degrees of freedom and a non-centrality parameter $\delta = -\lambda$. The t-distribution is used to capture the fat-tail distribution in the finance literature. Based on the obsevervation of Figure 3.3, we can notice that it is similar to Figure ??, the mixture of the non- central t-distribution of parameter $\theta = -\lambda$ with the Student-t distribution distorts the probability density distribution exactly as the 1-factor transform. The real contribution of Kijima (2006) is the extension of this transform and the Wang transform to a multivariate setting by using t-copula in order to preserve the linearity for the pricing functional, since the risk premium parameter $\lambda$ is no longer linear. Kijima & Muromachi (2006) describe the essence of the new two parameter transform (Wang transform extension) proposes by Kijima (2006) and show some special example related to the Student-t distributions.

A further extension of pricing contingent claims with probability distortion operator is the approach used by Godin *et al.* (2012). Godin *et al.* (2012) introduce a new distortion operator based on the Normal Inverse Gaussian distribution. In fact the Normal Inverse Gaussian (NIG), is a generalization of the normal distribution that allows for heavier skewed tails. The resulting operator asymmetrically distorts the underlying distribution. Godin *et al.* (2012) also show that it is possible to recuperate Non-Gaussian Black-Scholes formulas using their distortion operators.

$$g_{\alpha,\beta,\eta,\theta}(F(x)) = \Phi^{NIG}(\Phi^{NIG-1}(F(x)) + \theta) \tag{3.8}$$

With $\alpha, \beta, \eta$ representing the parameters of the Normal Inverse Gaussian cumulative distribution $\Phi^{NIG}$, $\theta$ the risk adjustment parameter, and $g$ the distortion operator.
Osu & Achi (2013) present the distortion proposed by Godin *et al.* (2012) and use distortion operators under a simple transformation to price contingent claim with a Cauchy distribution.

## 3.2 General pricing methods

Apart of the pricing methods involving probability distortion operators, there are other methods used for pricing CAT bond. These methods are based on financial, actuarial and econometric principles. Among these methods, there are approaches which apply extreme value theory to CAT bond pricing and use multiple of average expected loss to compute the required spread over the LIBOR which is the CAT bond yield. The outcomes of this approach are not always consistent with the observed CAT bond price. Other approaches determine the CAT bond price based on the expected frequency and the severity of the losses. the parameter is estimated based on the observed CAT bond price[3].

Lane (2000) presented the framework of pricing CAT bonds with the financial approach: the 3-parameter model. He shows that since the gross price of CAT bonds issued

[3]http://insuranceplanet.blogspot.it.

at par is expressed as the coupon accruing to investors, the CAT bond price will be composed of a part-financing risk equal to the LIBOR and a part insurance risk equal to the spread over the LIBOR. The spread is the sum of the expected losses (EL) which represent the investors compensation for his expected losses and the risk load which is also the unexpected loss. The unexpected loss is usually approximated by the standard deviation of the loss distribution only if the distribution is symmetric. Since CAT bond loss distribution is asymmetric, the author decomposes the unexpected loss component of the spread as the expected excess return (EER). The expected excess return (EER) or risk premium is the amount which an investor requires to commit the risk capital. The EER is a Cobb-Douglas function type of the conditional expected losses (CEL)[4] which capture the asymmetrical nature of the losses and the probability of first loss (PFL)[5]. In fact the CEL represent the severity of the losses, it is the amount of loss in term of capital in case of a first loss. It captures the riskiness of the bond, and it is more concentrated on the right tail of the loss curve. Lane (2000) developed the following model:

$$EER = \gamma(PFL)^{\alpha}(CEL)^{\beta}, \tag{3.9}$$

$$EL = PFL \times CEL. \tag{3.10}$$

By running some econometric regressions and p-value tests on the results of equation (3.10), the author obtains the best fit for parameters $\gamma$= 0.55, $\alpha$=0.495 and $\beta$=0.574. The spread $S$ is then written as follow:

$$S = EL + EER \tag{3.11}$$

$$S = EL + \gamma(PFL)^{\alpha}(CEL)^{\beta} \tag{3.12}$$

$$S = PFL.CEL + \gamma(PFL)^{\alpha}(CEL)^{\beta} \tag{3.13}$$

The price of the CAT bonds will be LIBOR plus the spread (S). Lane (2000) applies this method on a series of CAT bond issues in order to compute their yield. Lane (2003) also gives a more detailed explanation of this model introduce by Lane (2000). This

[4]CEL also represents the severity of losses

[5]PFL is defined by rating agencies

model is the first model which has been developed to understand the behaviour of CAT bond market; it is the unique model which tries to link the obtained results on CAT bond market and those obtained in the reinsurance market(Gatumel & Guégan 2009).

Cox & Pedersen (2000) presented the actuarial methodology for the valuation of catastrophe risk and developed a framework of pricing CAT bonds in incomplete market settings. In 2003 Burnecki & Kukla applied the result of Baryshnikov & al (2001) to calculate no-arbitrage prices of a zero-coupon and coupon CAT bonds, and derived the formula under the compound doubly stochastic Poisson framework model. Vaugirard (2003) uses the jump-diffusion model of Merton (1976) to develop the first valuation model of insurance-linked securities that deal with catastrophic events and interest rate randomness. Burnecki (2005) evaluated CAT bonds using a compound non homogeneous Poisson model with left truncated loss distribution. Härdle & Cabrera (2010) examine the calibration of real parametric CAT bonds for earthquakes sponsored by the Mexican government, using the results of Burnecki (2005). Jarrow (2010) develops a simple closed form solution for valuing CAT bonds, while the formula is consistent with any arbitrage-free model for the evolution of the LIBOR term structure of interest rates. Nowak & Romaniuk (2013) prove a general pricing formula which can be applied to CAT bonds with different payouts functions under the assumption of different models of risk-free spot interest rate.They price CAT bonds with interest rate dynamics describe by CIR[6] and Hull White model and use the Monte-Carlo simulation to analyse the numerical properties of the pricing formula obtained. Most prior studies did not take into account diverse factors that affect bond prices. Ma & Ma (2013) consider a variety of factors that affect bond prices such as loss severity distribution, claim arrival intensity, threshold level and interest rate uncertainty. Consequently, they derive a pricing formula for CAT bonds in a stochastic interest rate environment and show that the loss process follows a compound non homogeneous Poisson process.

In contrast to the above approaches used to price CAT bonds, there are not so much studies, which use the econometric method to price CAT bonds. Lane & Mahul (2008) examine 250 catastrophe bonds issued between 1997 and 2008 in order to determine the variables influence the value of the CAT bond's price. By applying a simple linear

[6]Cox-Ingersoll-Ross.

regression model, their analysis reveals that CAT bond prices are function of the underlying peril, the expected loss, and the reinsurance cycle. Ahrens *et al.* (2009) develop an econometric pricing model for CAT bonds with the aim to examine the impact of the year 2005 hurricanes season particularly hurricane Katrina on CAT bond's price. Their theoretical framework is based on the Lane Financial (LFC) model presented by Lane (2000). The results of the treed Bayesian estimation confirm that the severity component of the spread has an increased impact indicating a shift in investor perception during the pricing process. The results also show a significant increase of the impact of conditional expected loss through its interaction with the attachment probability. Ahrens *et al.* (2009) show that the influence of the conditional expected loss is increasing by investment grade rating because investors who demand highly rated bonds may be more concerned by possible losses than junk bond. Papachristou (2011) uses a dataset of 192 CAT bonds launched between 2003 and 2008, he applies a generalized additive model to examine the factors that affect the CAT bond risk premium. Gatumel & Guégan (2009) present the methodology approach of Wang (2004), Lane (2000) and Fermat Capital Management 2005, use to price Insurance-Linked-Securities. Using the three approaches, Gatumel & Guégan (2009) conduct a dynamical study of few CAT bond on the secondary markets from 2004 to 2009 in order to understand the elements driving the spread. The authors highlight both a structural component, the risk aversion of investors and the conjonctural component driving the spread. Their results show that some risk like US hurricanes, Europeans windstorm or California earthquakes impact the market significantly. Gürtler *et al.* (2014), by the means of panel data methodology, assess the effect of a natural catastrophe or financial crises on CAT bond premium. They find evidence that both the financial crisis and the hurricane Katrina significantly affected the CAT bond premiums. Their results also show a positive relationship between corporate bond spread and CAT bond premium which is not very consistent with the "Zero-beta" property of CAT bonds. Braun (2015), from a compile dataset of all CAT bonds issued between June 1997 and December 2012, develop a new econometric pricing model for CAT bonds in the primary market that is applicable across territories, perils and trigger type. Braun (2015) runs a series of OLS regressions with heteroscedasticity and autocorrelation consistent two standard errors aiming to identify the main drivers of CAT bond spreads. As Lane &

Mahul (2008) and Ahrens *et al.* (2009), he found that expected loss is the most important driver of the spread. Other factors like the covered territory, the sponsor, the reinsurance cycle and the spread on comparably rated corporate bonds also significantly impact CAT bond spread.

# Chapter 4

# The Wang transform application

When pricing CAT bonds, the main issue is to compute the premium paid by the sponsor to the SPV to be covered from the catastrophe risk. This premium corresponds to the alternative reinsurance price. It also represents the spread over the LIBOR which the SPV has to pay to the investors in case of no catastrophe event Lane (2000). This section is devoted to the development of the pricing based on distortion premium calculation principle and his possible applications to CAT bonds primary market data.

## 4.1 Premium calculation model

The model presented for the determination of the risk premium is the one developed by (Wang 2004). According to Galeotti *et al.* (2013), the Wang transform is one of the most successful techniques used for cat bonds pricing. Let assume the losses of a catastrophe event are defined by a non-negative random variable $X$. For the alternative reinsurance, the sponsor pays a premium $\Pi$ to the SPV to cover his losses up to a limit $h$. The limit is important for the reinsurer because losses from a catastrophe event are usually unlimited and unlimited losses cannot be covered. The loss $X$ is between zero and a maximum amount of losses $X_{max}$ ($X \in (0, X_{max}]$) with $X_{max} < \infty$.

In insurance, insured risk are usually divided in many layers or levels of risk. For a given contract we will have a range of layers $(b_i, b_i + h_i]$, with $i = 1, \ldots, n$ and $\cup_{i=1}^{n}(b_i, b_i + h_i] = (0, X_{max}]$. According to Froot (2001), reinsurance purchase should prioritize the highest layers which are associated with the most severe event. Following this idea, the layer for catastrophe risk will be $(b_n, b_n + h_n]$. So Wang (2004) define the loss of a CAT bond with an attachment[1] point $b$ and a limit $h$ as follow:

$$X_{[b,b+h]} = \begin{cases} 0 & \text{if X < b} \\ X - b & \text{if b} \leq \text{X < b+h} \\ h & \text{if X} \geq \text{b+h} \end{cases} \tag{4.1}$$

If the loss is less than the attachment point, there occurs no loss for the layer. For a loss $X$ comprises between the attachment point and the exhaustion point[2], the layer loss will be $X - b$ ,and $h$ for the case where the loss exceed the exhaustion point. The insurer expected loss for the layer $(b, b + h]$ will be the area below the loss exceedance curve. Since the losses are considered like a liabilities assets, the loss exceedance curve will the define by a survival function or a decumulative distribution function $S_X(x) = 1 - F_X(x)$ where $F_X(x)$ is the loss cumulative distribution function. Using the historical losses data, it is possible to determine the distribution of $S(x)$ or $F(x)$. Studies like Ma & Ma (2013) and Nowak & Romaniuk (2013) use the historical catastrophe losses to determine the loss distribution function. They tested many distribution like the lognormal distribution, the Burr, Weibull, Pareto and the Generalized Extreme Value distribution (GEV). Straßburger (2007) presents the some of the main geophysical commercial model[3] used by reinsurance companies to determine the decumulative distribution function and the exposure of their assets. For a given amount of loss $y$, we will have :

$$S_X[b, b + h](y) = \begin{cases} S_X(b + y) = Pr(X < b + y) & \text{if b < y < b+h} \\ 0 & \text{if y} \geq \text{h} \end{cases} \tag{4.2}$$

The premium $\Pi$ will be equal to the expected loss plus a safety loading or risk load, because the CAT bond market is an incomplete market (Cox & Pedersen 2000). Thus the expected loss will correspond to the risk neutral valuation and the risk loading will

[1] The attachment point is the amount of loss above which the layer register a loss.

[2] The exhaustion point represents the maximum amount of loss of the layer. In other word it is the maximum amount of loss that the insurer can cover

[3] The main geophysical model are divide in two categories: the property one which include the Risk Management Solutions(RMS), the Applied Insurance Research Worldwide (AIR) and the EQUECAT and the HAZUS which is and open-source model.

take into account the incompleteness of the market. So the expected value of the loss associated to the layer $(b, b+h)$ will be define as follows :

$$E[X] = \int_0^{+\infty} S_X(x)dx, \tag{4.3}$$
$$E[X_{[b,b+h]}] = \int_b^{b+h} S_X(x)dx. \tag{4.4}$$

Wang (2004) shows that if the limit $h$ is very small, the expected value loss of the layer will be equal to:

$$E[X_{[b,b+h]}] \approx hS(b).$$

The expected loss rate will be also equal to:

$$EL = \frac{E[X_{[b,b+h]}]}{h}. \tag{4.5}$$

Wang (2004) shows how to use the the Wang transform to compute the premium $\Pi$. Wang (2000) presented the new form of distortion operator $g(\cdot)$. $g(\cdot)$ function is increasing concave and defined on $[0,1]$ such that $g(0) = 0$ and $g(1) = 1$ with $g'(w) \leq 0$ [4] for the survival function to keep its characteristics and $g''(w) \geq 0$ to guarantee a non negative risk load. In fact, Wang (2004) shows that the Wang transform applied on the loss exceedance curve produces a risk ajusted loss exceedance curve $S_X^*(x)$,

$$S_X^*(x) = g(S_X(x))$$
$$S_X^*(x) = \Phi(\Phi^{-1}(S_X(x)) + \lambda)$$

In order to take in account the skewness of the catastrophe losses distributions and the parameters,Wang (2004) replace the normal distribution by the Student-t distribution $Q$ of $k$ degree of freedom.This new transform is the 2-factor Wang transform.

$$S_X^*(x) = Q(\Phi^{-1}(S_X(x)) + \lambda).$$

[4] For $0 < w < 1$.

According to Wang (2000) the parameter $\lambda$ ( $0 < \lambda < 1$) corresponds to the market price of risk or Sharpe ratio if the losses $X$ are normally distributed. In that case of an alternative distribution of $X$, the parameter is an extension of the Sharpe ratio (Wang 2004). Wang (2004) also justifies the usage of the Student-t adjustment as a way to capture the two opposing forces which often distort investors rational behaviour namely the "greed and fear". The fear of large unexpected losses is one of the investors main concern, but investors also desire unexpected large gains. So the magnitude of the distortion operator normally increased at the both extreme tails of probability distribution. The mean value under $S^*(x)$, denoted by $E^*[X]$ will represent the a risk adjusted fair value[5]of $X$ .

$$E^*[X_{(b,b+h)}] = \int_b^{b+h} g(S_X(x))dx = \int_b^{b+h} S_X^*(x)dx. \tag{4.6}$$

$E^*[X]$ contains already the risk loading (Wang 2004). . Since $E^*[X_{(b,b+h)}]$ is the transform expected value of the absolute layer loss $X_{(b,b+h)}$, following the same idea of equation (4.5), the transform expected loss (rate) will be:

$$EL^* = \frac{E^*[X_{(b,b+h)}]}{h}. \tag{4.7}$$

Based on equation (4.5) we can derive according to Wang (2004), the expression of the risk adjusted premium under the Wang transform:

$$\Pi(X) = EL^* = \frac{1}{h}\int_b^{b+h} g(S_X(x))dx. \tag{4.8}$$

Due to the absence of information on the distribution of $S_X(x)$ the survival function, we will need to introduce some variables in other to be able to compute the premium. We have the probability of first loss (PFL) equal to: $PFL = S(b) = P(X > b)$, the probability of exhaustion[6] (PE) defined by $PE = S(b+h) = P(X \leq b+h)$. Finally the conditional expected loss or the expected loss given default:

$$CEL = \frac{E^*[X_{(b,b+h)}|X > b]}{h} = \frac{EL}{PFL}$$

[5]**?**, page 21.

[6]The probability of exhaustion also called the probability of last loss represents the probability that the losses exceed the exhaustion point.

, Wang (2004) uses the piecewise linear interpolation to construct the loss decumulative distribution $S(x)$ and derive under the Wang transform $S^*(x)$. The determination of the premium $\Pi$ given the informations available requires the utilization of the trapezium rule:

$$\Pi(X) = \frac{1}{h}\int_b^{b+h} g(S_X(x))dx,$$
$$\Pi(X) \approx \frac{1}{h}\frac{1}{2}h[g(b) + g(b+h)],$$
$$\Pi(X) \approx \frac{1}{2}[g(b) + g(b+h)].$$

The expected loss under the Wang transform ($EL^*$) already incorporates the spread. In other to have the premium of the alternative reinsurance contract, we have to extract the expected loss ($EL$) computed under $S(x)$ Wang (2004) . So the final "premium" will be equal to:

$$\Pi_w(X) \approx \frac{1}{2}[g(PFL) + g(PE)] - PFL \cdot CEL. \tag{4.9}$$

According to Wang (2004), the 1-factor transform does not fit well for the pricing of CAT bonds. So in order to determine the parameters $k$ and $\lambda$ under the 2-factor Wang transform, we will estimate the following non linear model:

$$\Pi_w(X) = \frac{1}{2}[Q_k(\Phi^{-1}(PFL) + \lambda) + Q_k(\Phi^{-1}(PE) + \lambda)] - PFL \cdot CEL + \varepsilon. \tag{4.10}$$

with $\Pi_w(X)$ the premium of the CAT bond $X$ computed under the 2-factor Wang transform where $PFL$, $PE$, and $CEL$ represent respectively the probability of first loss, the probability of exhaustion of the CAT bond $X$ given by the primary market and $\varepsilon$ an error term. Since we do not have the transform premium $\Pi_w(X)$, in other to calibrate the parameter $\lambda$, we will use the premium (spread over the LIBOR) given by the market each. The computation procedure of the parameters $k$ and $\lambda$ and the results will be presented in the next sections.

## 4.2 Calibration and Results

The determination of the parameters $k$ and $\lambda$ of the 2-factor Wang transform will be done by using calibration methods. The remainder of this sector will be devoted to the presentation of our dataset, the calibration procedure and the comparison of the results obtain with the 2 factor Wang transform and Lane (2000) model.

### 4.2.1 Data and Methodology

#### Data

The absence of a publicly available database of CAT Bonds transactions represents an obstacle to the research on CAT bonds market. This absence can be justified by the fact that the CAT bonds market is a very recent market. Diverse data sources like Lane financial LLC, Artemis Deals directory are used in the literature to overcome this obstacle. For our empirical analysis, we used hand collected primary market data from Lane Financial LLC quarterly report on CAT bonds transactions that we cross-checked informations available on Artemis Deals Directory website. The dataset is composed of 69 CAT bonds issued from April 1, 2014 to March 30, 2016. For each CAT bond transaction we have the following informations: the probability of first loss (PFL), the probability of exhaustion (PE), the expected loss (EL) and the market spread over the LIBOR ($\Pi_m$). We also have some CAT bonds specific informations such as the issuer, the maturity, the trigger mechanism, and the rating. As presented in our model the only data needed for pricing CAT bonds under the Wang transform are the PFL, PE and the CEL. We will divide our data in two different sample: an in-sample composed of CAT bonds issued between April 1st, 2014 to March 31, 2015 and an out-of-sample. The in-sample period data is composed of 35 CAT bonds and the out-of-sample period data of 34 CAT bonds.

#### Calibration methodology

Calibration in finance can be defined as optimization method that consist in finding the set of model parameters that minimizes the difference between the model prediction and the available market data. In other words, calibrating a CAT bond pricing model under the 2-factor Wang transform means looking for the risk adjusted parameter such that the model premiums are consistent with the market premiums. The calibration will lead to an optimization problem where we will need to define the *error metric* that will measure the difference between the market premium ($\Pi_m$) and the 2-factor Wang transform model premium. In most cases, the objective function or the error metric defines the optimization problems as a minimisation task. The best fit parameter will be the one minimising the error metric. Many error metrics can be used :

- the sum of absolute relative deviation;

$$ARDev = \sum_{i=1}^{N} \frac{|\Pi_{wi} - \Pi_{mi}|}{\Pi_{mi}},$$
$$ADev = \sum_{i=1}^{N} |\Pi_{wi} - \Pi_{mi}|,$$

- the sum of the square deviation

$$SDev = \sum_{i=1}^{N} (\Pi_{wi} - \Pi_{mi})^2.$$

It is also possible to minimize the sum of the square root errors. All this error metrics produce approximately the same result. In this work, we will use the mean square error as the error metric to be minimized.

Calibrating CAT bond pricing model under the 2 factor Wang transform to the market premium lead to an optimization problem that cannot be solved with the standard methods such of the gradient methods. The calibration of the 2-factor Wang transform model requires to find the value of one parameter under a mixture of a Student-t and an inverse normal cumulative distribution function. So the objective function here is non linear.Even thought the standard optimization approach seems to be convenient for this type of the minimum search, finding the value of the risk adjusted parameter which makes the model consistent with the market premium implies solving a non-convex optimisation problem. In this case it is not possible to use a normal zero-finding optimization method. The standard approach used for non-linear model like the non-linear least square will give wrong results if applied form the calibration of the 2-factor Wang transform. This type of optimisation problem is common in option pricing, Gilli & Schumann (2011) show that the calibration of the Heston's stochastic volatility model and the Bates option model faces the same problem. The authors emphasize that the optimization problem is due to the non convexity of the objective function of those models. In the fact because the objective function presents many local minima, the direct search method could be trapped in a local minimum instead of the global minimum.

Since this type of models cannot be calibrated by the standard optimization methods, some alternative algorithms have been developed. Some of the algorithms developed to overcome this problem called the heuristic methods are: the downhill simplex, the Levenberg-Marquardt algorithm and the differential evolution. These three models are part of the model used by FINCAD analytics for the calibration of financial models. Storn & Price (1997) briefly present the Levenberg-Marquardt algorithm and the downhill simplex method and their drawbacks. The Levenberg-Marquardt algorithm is a technique used to solve non-linear optimization problem. It uses a combination of the gradient descent and the Gauss method to search the parameter that minimizes the objective function. The Downhill simplex also called the Nelder Mead method is a method that uses the concept of simplex to find the maximum or the minimum of an objective function. The downhill algorithm and the Levenberg-Marquardt algorithm both begin their search of the parameter by fixing and initial value of the parameter . In fact, they find the best value by trying to shift the current parameter towards the smaller value of the error metric. The parameter shift is downhill. The algorithm ends when the downhill shift cannot be achieved for the current parameter. A drawback of these methods is the dependence of the results on the initial value set for the parameter. For example the Levenberg-Marquardt algorithm will never find the global minimum if the initial value is no set near to it. So it is the determination of the global minimum with this approach is uncertain. For the downhill algorithm even thought it was possible to obtain the global minimum for any initial value, there is no guarantee that the search does not end at the local minimum. Therefore, the efficiency of the two approaches depends on the initial guess. The initialisation of a proper guess will lead to a successful calibration whereas a bad guess will give spurious results.

The differential evolution algorithm developed by Storn & Price (1997) is a solution of the initialization problem of guess initialization. The calibration result does not depend on the initial guess. Storn & Price (1997) present some requirements that a practical minimisation technique has to fulfill. The method has to be able to handle non-differentiable and non-linear cost function; it has to have fewer controls variable to steer the minimisation. The control variables have to also robust and easy to choose. the technique most have a good convergence properties, that means consistent convergence to the global minimum in consecutive independent trials. The differential evolution (DE) algorithm fulfils all these requirements. The DE was designed to be a stochastic direct search method. Direct search methods also have the advantage of being easily applied to experimental minimization where the cost value is derived from a physical experiment rather than a computer simulation (Storn & Price 1997). The DE is a parallel direct search method, instead of a single initial guess value, the DE algorithm evolves many trials value in parallel. Thus the DE scans the entire parameter space and when run enough, it virtually guarantees to find the global minimum. The only potential drawback of the differential evolution is runtime of the algorithm (Storn & Price 1997). The fact that the DE samples the entire parameter space makes that more iterations are required than for the Levenberg-Marquardt or the downhill simplex algorithm.

Figure 4.1: A typical evolutionary scheme of the Differential Evolution algorithm

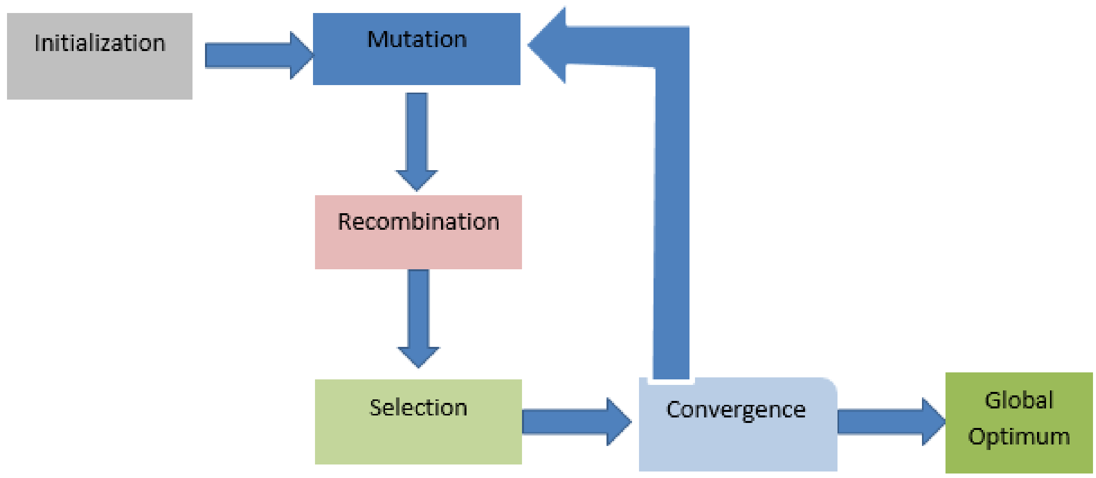


Figure 4.1 presents a typical evolutionary scheme of the differential evolution algorithm. The first step consists to initialize the parameter, then the mutation enables the expansion of the parameter search space. The recombination reuses the previously successful parameter; here there is a mixture successful solution from the previous generation with the current generated solution. At the selection stage, a minimum parameter is choice, if this parameter does not minimize the objective function, the model will not converge. In that case, the process restarts at the mutation step until the selected local minimum also corresponds to the global minimum. A greedy scheme is the key for fast convergence of differential evolution.

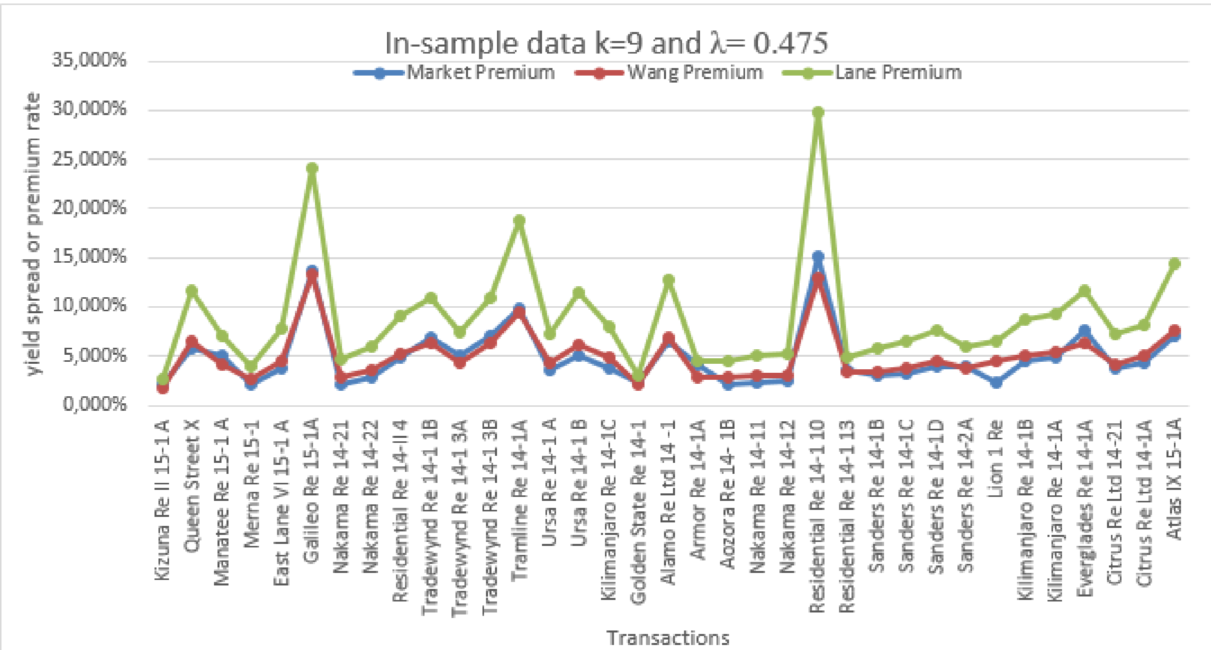
For the calibration of the risk adjusted parameter $\lambda$ and the degree of freedom $k$ , we will use the differential evolution algorithm. The best fit parameters $k$ and $\lambda$ determined for the 2-factor Wang transform will be the optimal parameter of the DE algorithm. We will use the optimal parameter to compute the premium under the 2-factor Wang transform, plot it and compare the distribution of the actual price given by the market. In other to determine the risk adjusted parameter $\lambda$ and the parameter $k$, we use the mean square errors as our objective function (error metric). The minimization of the objective function is performed with the differential evolution algorithm. Based on the results of Wang (2004) and Galeotti *et al.* (2013) which obtain respectively a $k$ parameter equal to 5 and 7 for CAT bonds, we set parameter search space such as $k \in [1, 9]$ with $k \in \mathbb{N}$. The risk ajusted parameter $\lambda$ is comprise in the in $[0, 1]$ so we initialize the parameter as continuous uniform random variable. The number of iterations is 1000.

1. Set the space of length $K$ of the parameter $k$, the benchmark error metric $mse = 10^3$, the number of iterations (IT), the vector collecting the $\lambda$'s, set the length of the data equation to N
2. **for** $j = 1$ to $K$ **do**
3. **for** $i$= 1 to $IT$ **do**
4. $\lambda_{(i,j)} = rand(1)$ .set the space:
5. $\lambda_1 = \lambda_{(i,j)}$
6. $SumSerr(j) = 0$ initialize the sum of square error
7. **for** $n = 1$ to $N$ **do**
8. $\Pi_{w(n)} = \frac{1}{2}[Q_{k(j)}((\Phi^{-1}(PFL(n))+\lambda_1)+[Q_{k(j)}(\Phi^{-1}(PE(n))+\lambda_1)]-CEL(n)\times PFL(n)$
9. $Serr = (\Pi_{w(n)} - \Pi_{m(n)})^2$
10. $SumSerr(j) = SumSerr(j) + S_e rr$
11. **end for**
12. $Cmse(i,j) = SumSerr_{(j)]}.\frac{1}{N}$ compute the error metric (sum mean square error);
13. **if** $Cmse(i,j) < mse_{(j)}$ **do** selected the minimum mean square error;
14. $mse_{(j)} = Cmse_{(i,j)}$
15. $\lambda_{optimal(j)} = \lambda_{(i,j)}$ vector of every optimal $\lambda$ for a given $k$.
16. **end if**
17. **end for**
18. **end for**

### 4.2.2 Results and Implications

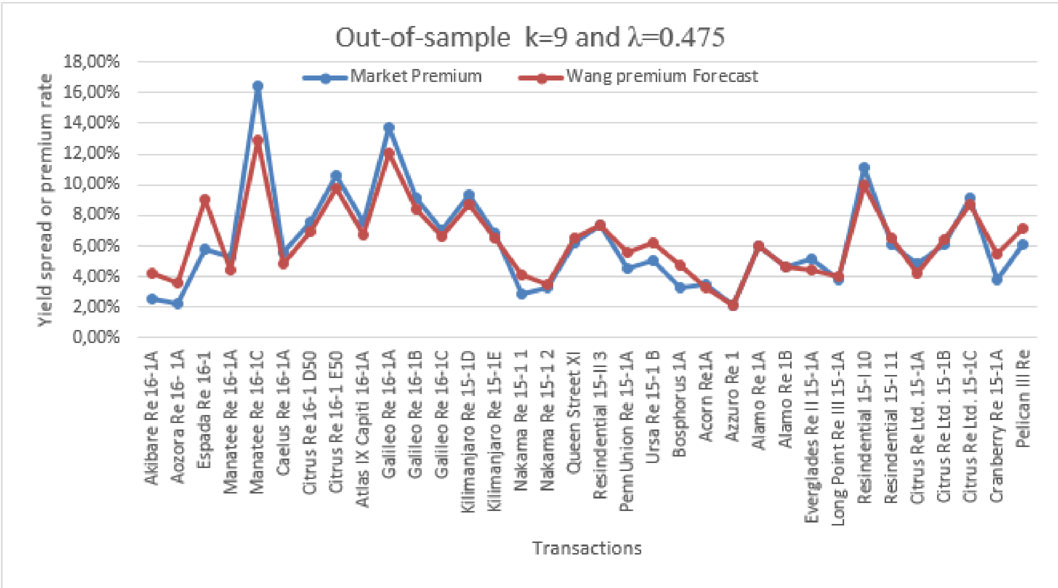
We apply the distortion approach to the primary market data of 35 CAT bonds issued between April 2014 and March 2015, which represents our in-sample period. The best fit parameters that minimize the mean square error are: $\lambda = 0.475$ for the risk adjusted parameter and $k = 9$ for the Student-t degrees of freedom parameter. We also compute the premium based on Lane (2000) pricing approach for the in-sample. Figure (4.2) shows the fitting results of the 2-factor Wang transform premium and the Lane premium for 35 CAT bonds of our in-sample data. We can see that the Wang transform premium is consistent with the market price while the difference between the Lane model premium and the market is very large. The mean absolute relative deviation for the 2-factor Wang transform model is equal to 0.1 whilst it is equal to 0.57 for the Lane model. The comparison of this error metric for the two model confirms the difference between the premium of two model as compare to the market premium.

Figure 4.2: Fit of the 2-factor Wang Transform to market yields spreads and Lane model yields spreads for 35 CAT bonds transactions data between April 1, 2014 to March 31,2015.



We use the fitted parameters on the out-of-sample data and we can see that the 2-factor Wang transform can reasonably explain the premium given by the market (see Figure (4.3)). The 2-factor Wang transform model premium is very consistent with the market premium when using the calibrated parameter of the in-sample data to price the out- of-sample CAT bonds, We have an error metric of 0.2.

Figure 4.3: Using the in-sample fitted parameters ($\lambda$ = 0.475 and $k$ = 9 ) to test the market yields spreads for 34 CAT bonds transactions data from April 1, 2015 to March 31, 2016.




After computing the parameters of the 2-factor Wang transform model for the in-sample, we use the differential algorithm to compute the parameters of the model for the out-of-sample data composed of 34 CAT bonds transactions form April 1, 2015 to March 31, 2016. We want to observe if there is a significant change between the parameters of the in-sample and those of the out-of sample data, since the two samples contain slightly the same number of CAT bonds transactions. The result of the differential evolution gives the best fitted parameters with $k = 9$ for the t-student degree of freedom and the risk-adjustment parameter $\lambda = 0.49$. We can observe that the parameter $k$ does not change but there is a change of 0.015 for the risk-adjusted parameter. Figure (4.4) shows that the yields spread from our model compute for the out-of -sample data is very consistent with the market yields spreads. The yields spreads computed with Lane (2000) is not very consistent with the market yields spreads as compare to the 2-factor Wang transform model results. We compute the average absolute relative deviation for the two models and we obtain repectively 0.141 and 0.82 for the 2-factor Wang transform model and Lane (2000) model.

Figure 4.4: Fit 2-factor Wang transform to market yields spreads and Lane model yields spreads for 34 CAT bonds transactions data between April 1, 2015 and March 31, 2016 (Fitted parameter $k = 9$ and $\lambda = 0.49$).

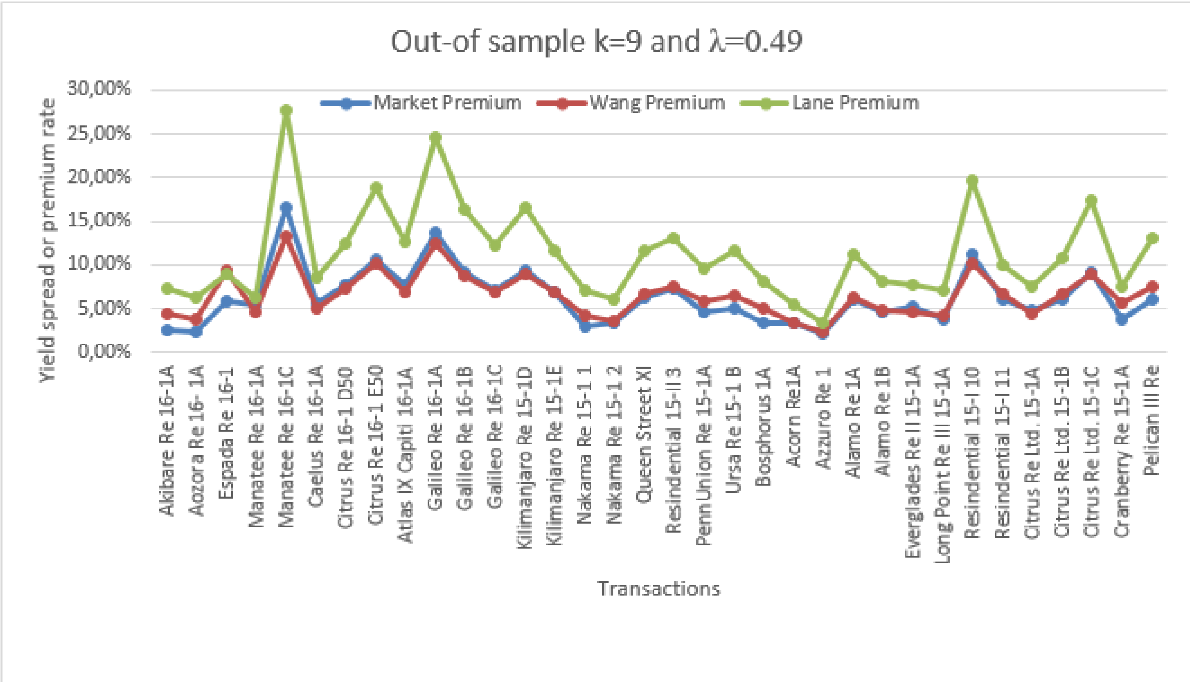



Table 4.1: Comparison of the models based on the mean absolute relative error

| | In-sample | | Out-of-sample | |
|---|---|---|---|---|
| Model | 2-factor Wang Transform | Lane(2000) | 2-factor Wang transform | Lane (2000) |
| MAE | 0.1 | 0.57 | 0.14 | 0.82 |

The results obtained with the 2-factor Wang transform model for the in-sample and out-of- sample, and the Lane (2000) model confirm the conclusion of Galeotti *et al.* (2013) who considered the 2-factor Wang transform as the most accurate and the best model for CAT bonds pricing. The 2-factor Wang transform approximate very well the market spread of CAT bonds. The 2-factor Wang transform is a good tools, that market players can used to predict CAT bond price, since investors are usually provide with the PFL, PE and CEL for each transactions.

# Chapter 5

# Conclusion

This past 20 years, the world has registered an important and significant increase of the number of natural disasters, coupled with the effect of climate change and the increase of the population in regions with high threats of natural catastrophe. This situation increases the insolvency risk for insurance company due to the huge amount of losses related to the occurrence of a catastrophe event. The aim of this thesis was to present the catastrophe risk and his valuation framework. In order to do it, we first present the catastrophe risk, his properties and the instruments used by insurance companies to hedge this type of risk. Then we present the state of the art of the pricing methods of a catastrophe risk hedging instrument: CAT bond. Finally we focused on the 2-factor Wang transform pricing model which we compared the accuracy with the Lane (2000) model.

Catastrophe risk is a risk characterized by a low frequency and a high severity. This characteristic are the properties of a natural disaster. The Hurricane Andrew experience has shown that the normal reinsurance was not able to manage this type of risk. Insurance-linked securities like CAT bonds were launched to help insurance companies and sponsors to hedge the catastrophe risk and fulfill the condition of the equity capital requirement. It appears that CAT bonds have some similarity with credit default bonds; they are both high yield bonds. CAT bonds earn high yield because of the unpredictable characteristics of catastrophe events, while credit default bonds earn high yield because of their defaultable property. CAT bonds present the advantage to be fully collateralized; a property which covers CAT bond from interest rate risk and credit risk. The lack of transparency in the CAT bond market and the incompleteness make difficult to determine an accurate pricing model for CAT bond. We realise a state of the art of CAT bonds premium calculation and we notice that was a challenging issues.

We focused on the 2-factor Wang transform model which is one of the actuarial pricing approaches using probability distortion operators. We used data of the primary market for CAT bonds issued for April 1, 2014 to March 31, 2016. We divided our dataset in two sets: an in-sample composed of data from April 1, 2014 to March 31,2015, and an out-of-sample composed of data for April 1, 2015 to March 31,2016. We first determine the model parameter using the in-sample data. Secondly, we apply the calibrated model on the out-of -sample data. In order to evaluate the accuracy of our model, we also determine the parameter for the out-of-sample data. We compared the obtained results of the 2-factor Wang transform model both for the in-sample and the out-of-sample with the Lane (2000) model on the basis of the mean absolute relative error. The 2-factor Wang transform appears to be the most accurate model. We can conclude that 2-factor Wang transform is the most accurate model for CAT bond pricing. The 2-factor Wang transform model present the advantage that it takes into account the so-called greed and fear" behaviour of the investor, and that it need only the information on the PFL, PE and CEL to price a CAT bond.

Since there is no publicly available dataset of CAT bonds transactions, a good forward looking research will be to built a large dataset on issued CAT bonds transactions and determine the various characteristics of a CAT bonds that influence the investor perception of the CAT bond market. It will be also interested to analyse the accuracy of the 2-factor Wang transform and other probability distortion operator like the one presented by Godin *et al.* (2012) in a case of ambiguity aversion in the same framework presented by Robert & Therond (2014) and explore its possible extensions in credit risk valuation framework.

# Bibliography


Ahrens, Frieder, Füss, Roland, & Kestel, Sevtap. 2009. An econometric pricing model for CAT bonds and the impact of the 2005 hurricane season. *European Business School Research Paper*.

Bouriaux, Sylvie, & MacMinn, Richard. 2009. Securitization of catastrophe risk: New developments in insurance-linked securities and derivatives. *Journal of Insurance Issues*, 1–34.

Braun, Alexander. 2015. Pricing in the primary market for cat bonds: new empirical evidence. *Journal of Risk and Insurance*, DOI: 10.1111/jori.12067.

Burnecki, Krzysztof. 2005. Pricing catastrophe bonds in a compound non-homogeneous Poisson model with left-truncated loss distributions. *In: Mathematics in Finance Conference*.

Burnecki, Krzysztof, & Kukla, Grzegorz. 2003. Pricing of zero-coupon and coupon CAT bonds. *Applicationes Mathematicae*, **30**(3), 315–324.

Burnecki, Krzysztof, Kukla, Grzegorz, & Taylor, David. 2011. Pricing of catastrophe bonds. *Pages 371–391 of: Statistical Tools for Finance and Insurance*. Springer.

Cox, Samuel H, & Pedersen, Hal W. 2000. Catastrophe risk bonds. *North American Actuarial Journal*, **4**(4), 56–82.

Cummins, J David. 2008. CAT Bonds and Other Risk-Linked Securities: State of the Market and Recent Developments. *Risk Management and Insurance Review*, **11**(1), 23–47.

Cummins, J David, *et al.* 2006. Should the government provide insurance for catastrophes. *Federal Reserve Bank of St. Louis Review*, **88**(4), 337–379.

Division, United Nations. Population. 2002. *World urbanization prospects: the 2001 revision.* UN.

Froot, Kenneth A. 2001. The market for catastrophe risk: a clinical examination. *Journal of Financial Economics*, **60**(2), 529–571.

Galeotti, Marcello, Gürtler, Marc, & Winkelvos, Christine. 2013. Accuracy of premium calculation models for CAT bondsan empirical analysis. *Journal of Risk and Insurance*, **80**(2), 401–421.

Garg, Sandip. 2008. Multiyear Cat Bonds and Investor Behavior. *Wharton Research Scholars Journal.*

Gatumel, Mathieu, & Guégan, Dominique. 2009. Towards an understanding approach of the Insurance Linked Securities Market. *Available at SSRN 1924921.*

Gatzert, Nadine, Schmeiser, Hato, & Toplek, Denis. 2007. An analysis of pricing and basis risk for industry loss warranties. *Zeitschrift für die gesamte Versicherungswissenschaft*, 1–21.

Gerber, Hans U, Shiu, Elias SW, *et al.* 1994. Option pricing by Esscher transforms. *Transactions of the Society of Actuaries*, **46**(99), 140.

Gilli, Manfred, & Schumann, Enrico. 2011. Calibrating option pricing models with heuristics. *Pages 9–37 of: Natural Computing in Computational Finance.* Springer.

Godin, Frédéric, Mayoral, Silvia, & Morales, Manuel. 2012. Contingent claim pricing using a normal inverse Gaussian probability distortion operator. *Journal of Risk and Insurance*, **79**(3), 841–866.

Gürtler, Marc, Hibbeln, Martin, & Winkelvos, Christine. 2014. The impact of the financial crisis and natural catastrophes on CAT bonds. *Journal of Risk and Insurance.*

Hamada, Mahmoud, & Sherris, Michael. 2003. Contingent claim pricing using probability distortion operators: methods from insurance risk pricing and their relationship to financial theory. *Applied Mathematical Finance*, **10**(1), 19–47.

Härdle, Wolfgang Karl, & Cabrera, Brenda López. 2010. Calibrating CAT bonds for Mexican earthquakes. *Journal of Risk and Insurance*, **77**(3), 625–650.

Jarrow, Robert A. 2010. A simple robust model for CAT bond valuation. *Finance Research Letters*, **7**(2), 72–79.

Kijima, Masaaki. 2006. A multivariate extension of equilibrium pricing transforms: The multivariate Esscher and Wang transforms for pricing financial and insurance risks. *Astin Bulletin*, **36**(01), 269–283.

Kijima, Masaaki, & Muromachi, Yukio. 2006. On the Wang transform with fat-tail distributions. *In: Proceedings of Stanford–Tsukuba Workshop, Stanford University.*

Kijima, Masaaki, Motomiya, Shin-ichi, & Suzuki, Yoichi. 2010. Pricing of CDOs based on the multivariate Wang transform. *Journal of Economic Dynamics and Control*, **34**(11), 2245–2258.

Kijimaa, Masaaki, & Muromachia, Yukio. 2011. Extensions of the Wang transform for the pricing of insurance and financial risks.

Lane, Morton, & Mahul, Olivier. 2008. Catastrophe risk pricing: an empirical analysis. *World Bank Policy Research Working Paper Series, Vol (2008).*

Lane, Morton N. 2000. Pricing risk transfer transactions. *Astin Bulletin*, **30**(02), 259–293.

Lane, Morton N. 2003. Rationale and results with the LFC cat bond pricing model. *Lane Financial Ltd., Chicago.*

Litzenberger, Robert H, Beaglehole, David R, & Reynolds, Craig E. 1996. Assessing catastrophe reinsurance-linked securities as a new asset class. *The Journal of Portfolio Management*, **23**(5), 76–86.

Ma, Zong-Gang, & Ma, Chao-Qun. 2013. Pricing catastrophe risk bonds: A mixed approximation method. *Insurance: Mathematics and Economics*, **52**(2), 243–254.

McGhee, Christopher, Faust, Justin, & Clark, Ryan. 2005. The Growing Appetite for Catastrophe Risk: The Catastrophe Bond Market at Year-End 2004, Markt Report, Marsh & McLennan Securities Corporation und Guy Carpenter & Company. *Inc., New York*.

Nowak, Piotr, & Romaniuk, Maciej. 2013. Pricing and simulations of catastrophe bonds. *Insurance: Mathematics and Economics*, **52**(1), 18–28.

Osu, Bright O, & Achi, Godswill U. 2013. Contingent claim pricing using the Cauchy probability distortion operator under simple transformation. *International Journal of Applied Physics and Mathematics*, **3**(1), 8.

Ou Yang, Chieh. 2010. Managing catastrophic risk by alternative risk transfer instruments. *Publicly accessible Penn Dissertations. Paper 220.*

Papachristou, Dimitris. 2011. Statistiscal Analysis of the Spreads of Catastrophe Bonds at the Time of Issue. *Astin Bulletin*, **41**(01), 251–277.

Robert, Christian Y, & Therond, Pierre-E. 2014. Distortion risk measures, ambiguity aversion and optimal effort. *Astin Bulletin*, **44**(02), 277–302.

Storn, Rainer, & Price, Kenneth. 1997. Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces. *Journal of Global Optimization*, **11**(4), 341–359.

Straßburger, Doreen. 2007. *Risk management and solvency: mathematical methods in theory and practice*. Ph.D. thesis, Universität Oldenburg.

Swiss, Re. 2015. Natural catastrophes and man-made disasters in 2014: convective and winter storms generate most losses. *Sigma*.

Vaugirard, Victor E. 2003. Pricing catastrophe bonds by an arbitrage approach. *The Quarterly Review of Economics and Finance*, **43**(1), 119–132.

Wang, Shaun. 1995. Insurance pricing and increased limits ratemaking by proportional hazards transforms. *Insurance: Mathematics and Economics*, **17**(1), 43–54.

Wang, Shaun. 1996. Premium calculation by transforming the layer premium density. *Astin Bulletin*, **26**(01), 71–92.

Wang, Shaun. 2002a. Pricing of catastrophe bonds. *Alternative Risk Strategies*, 221–240.

Wang, Shaun S. 2000. A class of distortion operators for pricing financial and insurance risks. *Journal of risk and insurance*, 15–36.

Wang, Shaun S. 2002b. A universal framework for pricing financial and insurance risks. *Astin Bulletin*, **32**(02), 213–234.

Wang, Shaun S. 2004. Cat bond pricing using probability transforms. *Geneva Papers: Etudes et Dossiers, special issue on Insurance and the State of the Art in Cat Bond Pricing*, **278**, 19–29.

## .1 Appendix A

Table 1: Descriptives Statistics

| | Amount (millions $) | Market Premium(%) | PFL | PE | CAT Bond Maturity (month) | CEL |
|---|---|---|---|---|---|---|
| Mean | 210.84 | 5.5932 | 0.76648 | 0.017799 | 42.87 | 0.03345 |
| Standard deviation | 188.45 | 3.1840 | 0.14183 | 0.016928 | 9.8457 | 0.03397 |
| Sample Size | 69 CAT bonds transaction issued between April 1, 2014 and March 31, 2016 | | | | | |

Table 2: Results of the 2-factor Wang transform model and Lane model on the on the in-sample data

| SECURITIES | PFL | PE | CEL | Market yield | Wang yield | Lane yield |
|---|---|---|---|---|---|---|
| Kizuna Re II 15-1 A | 0,00210 | 0,0018 | 0,907 | 2,030% | 1,767% | 2,648% |
| Queen Street X | 0,03670 | 0.0203 | 0.741 | 5.830% | 6.561% | 11.738% |
| Manatee Re 15-1 A | 0.01590 | 0.0079 | 0.723 | 5.070% | 4.185% | 7.027% |
| Merna Re 15-1 | 0.00560 | 0.0032 | 0.732 | 2.030% | 2.595% | 3.941% |
| East Lane VI 15-1 A | 0.01450 | 0.0123 | 0.924 | 3.800% | 4.455% | 7.804% |
| Galileo Re 15-1A | 0.16680 | 0.0424 | 0.516 | 13.690% | 13.342% | 24.110% |
| Nakama Re 14-21 | 0.00590 | 0.0052 | 0.915 | 2.160% | 2.908% | 4.659% |
| Nakama Re 14-22 | 0.00910 | 0.0077 | 0.923 | 2.920% | 3.538% | 5.970% |
| Residential Re 14-II 4 | 0.02510 | 0.0117 | 0.713 | 4.870% | 5.184% | 9.099% |
| Tradewynd Re 14-1 1B | 0.03680 | 0.0156 | 0.655 | 6.840% | 6.307% | 10.824% |
| Tradewynd Re 14-1 3A | 0.01560 | 0.01 | 0.801 | 5.070% | 4.363% | 7.425% |
| Tradewynd Re 14-1 3B | 0.03680 | 0.0156 | 0.655 | 7.100% | 6.307% | 10.824% |
| Tramline Re 14-1A | 0.07470 | 0.0442 | 0.764 | 9.890% | 9.525% | 18.755% |
| Ursa Re 14-1 A | 0.01270 | 0.0112 | 0.929 | 3.550% | 4.225% | 7.253% |
| Ursa Re 14-1 B | 0.02810 | 0.0232 | 0.907 | 5.070% | 6.174% | 11.423% |
| Kilimanjaro Re 14-1C | 0.02260 | 0.0093 | 0.646 | 3.800% | 4.889% | 8.017% |
| Golden State Re 14-1 | 0.00490 | 0.0011 | 0.51 | 2.230% | 2.092% | 2.936% |
| Alamo Re Ltd 14 -1 | 0.04110 | 0.0231 | 0.752 | 6.440% | 6.947% | 12.710% |
| Armor Re 14-1A | 0.00670 | 0.0045 | 0.776 | 4.060% | 2.933% | 4.511% |
| Aozora Re 14- 1B | 0.00570 | 0.0049 | 0.912 | 2.030% | 2.838% | 4.561% |
| Nakama Re 14-11 | 0.00660 | 0.0059 | 0.955 | 2.280% | 3.059% | 5.093% |
| Nakama Re 14-12 | 0.00680 | 0.0061 | 0.956 | 2.530% | 3.106% | 5.182% |
| Residential Re 14-1 10 | 0.13530 | 0.0935 | 0.836 | 15.210% | 12.857% | 29.749% |
| Residential Re 14-1 13 | 0.01000 | 0.0044 | 0.63 | 3.550% | 3.313% | 4.947% |
| Sanders Re 14-1B | 0.00880 | 0.0071 | 0.898 | 3.040% | 3.447% | 5.757% |
| Sanders Re 14-1C | 0.01090 | 0.0088 | 0.89 | 3.300% | 3.839% | 6.463% |
| Sanders Re 14-1D | 0.01460 | 0.0118 | 0.877 | 3.950% | 4.459% | 7.575% |
| Sanders Re 14-2A | 0.01170 | 0.0065 | 0.752 | 3.950% | 3.676% | 6.045% |
| Lion 1 Re | 0.02320 | 0.0046 | 0.466 | 2.280% | 4.562% | 6.588% |
| Kilimanjaro Re 14-1B | 0.02420 | 0.0109 | 0.682 | 4.560% | 5.111% | 8.648% |
| Kilimanjaro Re 14-1A | 0.02550 | 0.0133 | 0.718 | 4.820% | 5.403% | 9.227% |
| Everglades Re 14-1A | 0.03340 | 0.0202 | 0.802 | 7.600% | 6.255% | 11.686% |
| Citrus Re Ltd 14-21 | 0.01300 | 0.0101 | 0.9 | 3.800% | 4.119% | 7.203% |
| Citrus Re Ltd 14-1A | 0.01910 | 0.013 | 0.78 | 4.310% | 4.971% | 8.212% |
| Atlas IX 15-1A | 0.04560 | 0.031 | 0.825 | 7.100% | 7.568% | 14.443% |

yied = premium

Table 3: Results of the 2-factor Wang transform model and Lane model on the on the out-of-sample data

| SECURITIES | PFL | PE | CEL | Market yield | Wang2 | Lane yield | Wang2 |
|---|---|---|---|---|---|---|---|
| Akibare Re 16-1A | 0.0136 | 0.0102 | 0.8750 | 2.53% | 4.32% | 7.26% | 4.193% |
| Aozora Re 16- 1A | 0.0107 | 0.0073 | 0.8410 | 2.23% | 3.75% | 6.17% | 3.647% |
| Espada Re 16-1 | 0.0833 | 0.0034 | 0.2320 | 5.83% | 9.28% | 8.88% | 9.068% |
| Manatee Re 16-1A | 0.0192 | 0.0060 | 0.5100 | 5.32% | 4.57% | 6.26% | 4.450% |
| Manatee Re 16-1C | 0.1428 | 0.0759 | 0.7230 | 16.48% | 13.35% | 27.75% | 12.949% |
| Caelus Re 16-1A | 0.0178 | 0.0140 | 0.8710 | 5.58% | 5.03% | 8.47% | 4.885% |
| Citrus Re 16-1 D50 | 0.0419 | 0.0220 | 0.7180 | 7.60% | 7.19% | 12.47% | 6.982% |
| Citrus Re 16-1 E50 | 0.0811 | 0.0419 | 0.7090 | 10.65% | 10.10% | 18.77% | 9.792% |
| Atlas IX Capiti 16-1A | 0.0360 | 0.0245 | 0.8330 | 7.60% | 6.90% | 12.55% | 6.692% |
| Galileo Re 16-1A | 0.1274 | 0.0590 | 0.6800 | 13.69% | 12.42% | 24.56% | 12.044% |
| Galileo Re 16-1B | 0.0590 | 0.0355 | 0.7750 | 9.13% | 8.72% | 16.28% | 8.450% |
| Galileo Re 16-1C | 0.0355 | 0.0231 | 0.8030 | 7.10% | 6.84% | 12.14% | 6.632% |
| Kilimanjaro Re 15-1D | 0.0625 | 0.0365 | 0.7540 | 9.38% | 8.99% | 16.57% | 8.720% |
| Kilimanjaro Re 15-1E | 0.0358 | 0.0210 | 0.7540 | 6.84% | 6.77% | 11.70% | 6.572% |
| Nakama Re 15-1 1 | 0.0131 | 0.0101 | 0.8850 | 2.91% | 4.26% | 7.16% | 4.143% |
| Nakama Re 15-1 2 | 0.0094 | 0.0075 | 0.9150 | 3.30% | 3.63% | 6.05% | 3.531% |
| Queen Street XI | 0.0362 | 0.0200 | 0.7400 | 6.24 % | 6.71% | 11.63% | 6.516% |
| Resindential 15-II 3 | 0.0475 | 0.0227 | 0.6860 | 7.35% | 7.57% | 13.06% | 7.346% |
| PennUnion Re 15-1A | 0.0258 | 0.0155 | 0.7440 | 4.56% | 5.80% | 9.51% | 5.633% |
| Ursa Re 15-1 B | 0.0289 | 0.0239 | 0.9070 | 5.07% | 6.46% | 11.62% | 6.266% |
| Bosphorus 1A | 0.0199 | 0.0109 | 0.7390 | 3.30% | 4.94% | 8.12% | 4.796% |
| Acorn Re1A | 0.0096 | 0.0052 | 0.7710 | 3.45% | 3.39% | 5.49% | 3.292% |
| Azzuro Re 1 | 0.0040 | 0.0022 | 0.7750 | 2.18% | 2.23% | 3.40% | 2.169% |
| Alamo Re 1A | 0.0274 | 0.0214 | 0.8980 | 5.98% | 6.17% | 11.17% | 5.981% |
| Alamo Re 1B | 0.0161 | 0.0130 | 0.8820 | 4.66% | 4.81% | 8.05% | 4.673% |
| Everglades Re II 15-1A | 0.0146 | 0.0119 | 0.8970 | 5.22% | 4.57% | 7.69% | 4.443% |
| Long Point Re III 15-1A | 0.0128 | 0.0095 | 0.8670 | 3.80% | 4.18% | 6.97% | 4.065% |
| Resindential 15-I 10 | 0.0833 | 0.0463 | 0.7440 | 11.15% | 10.26% | 19.76% | 9.946% |
| Resindential 15-I 11 | 0.0463 | 0.0089 | 0.4670 | 6.08% | 6.75% | 9.92% | 6.561% |
| Citrus Re Ltd. 15-1A | 0.0131 | 0.0114 | 0.9310 | 4.82% | 4.39% | 7.39% | 4.267% |
| Citrus Re Ltd. 15-1B | 0.0401 | 0.0144 | 0.6080 | 6.08% | 6.64% | 10.85% | 6.452% |
| Citrus Re Ltd. 15-1C | 0.0623 | 0.0401 | 0.8110 | 9.13% | 9.00% | 17.40% | 8.719% |
| Cranberry Re 15-1A | 0.0308 | 0.0070 | 0.4470 | 3.85% | 5.66% | 7.56% | 5.511% |
| Pelican III Re | 0.0444 | 0.0235 | 0.7270 | 6.08% | 7.39% | 13.03% | 7.174% |

Wang Premium with $\lambda = 0.49$ Wang2 = Wang Premium with $\lambda = 0.475$