Corso di Laurea magistrale
(*ordinamento ex D.M. 270/2004*)
in Economia e Finanza

Tesi di Laurea

Italian Electricity Prices: A Bayesian Approach

Relatore
Prof. Domenico Sartore
Prof. Roberto Casarin

Laureando
Michele Pettenò
Matricola 801457

Anno Accademico
2011 / 2012

Contents

1 Introduction

In the last decades the European directives contribute to the development of a liberalized market, in order to improve the electric system efficiency and reduce electricity prices.

In this scenario of liberalization, Gestore Mercato Elettrico became (since 1999) the company in charge of the economic organization and management of the Italian electricity wholesale market, commonly known as Italian Power Exchange (Ipex).

After five years of decrees and directives, the Italian power system underwent substantial reorganizations and it was divided into six electricity zones each one characterized by its own price.

The original contribution of the thesis is the evaluation of the zonal interdependences on the long run, in the new country energy market. Moreover we provide a multivariate analysis that takes into account determinant factors in electricity prices formation such as the weather changes and the presence of seasonalities.

The model is applied to equilibrium electricity spot prices of the Italian Wholesale Market and includes few groups of variables: lagged prices, weather, loads and periodic components.

As regard to the latter aspect, electricity may be considered as an atypical commodity due to its *non storability*. This peculiarity implies that generation and consumption of electricity have to be constantly balanced in real time. Hence, given these premises, the spot prices time series will exhibit different type of seasonalities which we will try to overcome using the Fourier Spectral Analysis as tool to identify them.

As regard to the weather conditions, we consider the air temperatures as a proxy that is able to capture habits and climate variabilities across different countries or regions. We will pay special attention to modelling the relationship between the behavior of the temperatures and the different zonal prices.

Another original contribution is given by the Bayesian approach to inference that allow us to avoid the problem of over-fitting.

This thesis is organized as follows. Chapter 2 discusses the Italian electricity system background and the zones in which it is divided. Chapter 3 shows a preliminary analysis of the time series that are included in the model. Chapter 4 gives an overview of the Bayesian methodology, proceeding with the description of the three priors that will be used to run the analysis. The results are given in Chapter 5 and Chapter 6 concludes.

2 The Italian Electricity Market

The Italian power system is organized like a grid: generation, transmission and distribution of energy are the core activities that let the energy system works properly, each one carried out by a different subject.

Ministero dello Sviluppo Economico (MSE) defines the strategic and operational goals considering the national interests, the *Authority of electric power* (AEEG) promotes competition and efficiency in the national electricity market, the responsibility of *Terna S.p.A.* is to guarantee the safety of the grid system that lets the electricity be transferred all over Italy.

The italian wholesale electricity market, better known as “Italian Power Exchange”, is managed by *Gestore Mercato Elettrico* which aims to boost the competition among the providers guaranteeing neutrality, transparency, objectivity in the country electricity market.

The market is divided into *the “Spot Electricity Market - MPE”, the “Forward Electricity Market - MTE”* subjected to the physical delivery of electricity and the *“Forward Electricity Account Trading Platform - PCE* “ on which the operators register their commercial obligations and assign the related injection and withdrawal of electricity.

Problems that have to be taken into account in the analysis are due to several factors such as the need to maintain a balanced amount of energy which gets into the network, because susceptible to losses during transport or distribution. Maintaining the frequency and voltage of the energy are two other key points in the efficiency maintenance of the plants.

Electricity is not a typical commodity because it cannot be stored. This unique characteristic makes complicated the compliance of the previews constrains.

Another factor to be borne in mind is the nature of the Italian market itself, especially if we compare it with the European ones. Therefore the market for GME is not a purely financial market aimed only to the determination of prices and quantities, but it is a real physical market where physical injection and withdrawal schedules are defined.

2.1 The Market Zones

The transmission of electricity within Italian territory takes place in geographic areas defined as *market zones*. These areas do not correspond to the Italian regions, but rather to an aggregate of them.

The regions that belong to the same market area pay the same price for the electricity. Each zone is characterized by limits in transmission of electricity to or from the neighboring zones. The range of the limit is defined by the balance between generation and consumption of energy and it is different among the macro-regions.

Figure 1: Italian Market Zones

The national transmission grid is interconnected with foreign countries through *18 lines*: four with France, nine with Switzerland, one with Austria, two with Slovenia, one submarine cable with Greece and one submarine cable between Sardinia and Corsica.

The shape of these areas, adopted by Terna, is functional to the management of the electric transit all along the peninsula and it can be summarized as follows:

- 6 *physical zones* (Northern Italy, Central Northern Italy, Central Southern Italy, Southern Italy, Sicilia and Sardinia);
- 6 *virtual zones* (France, Switzerland, Austria, Slovenia, Corsica and Greece);
- a series of interconnecting areas.

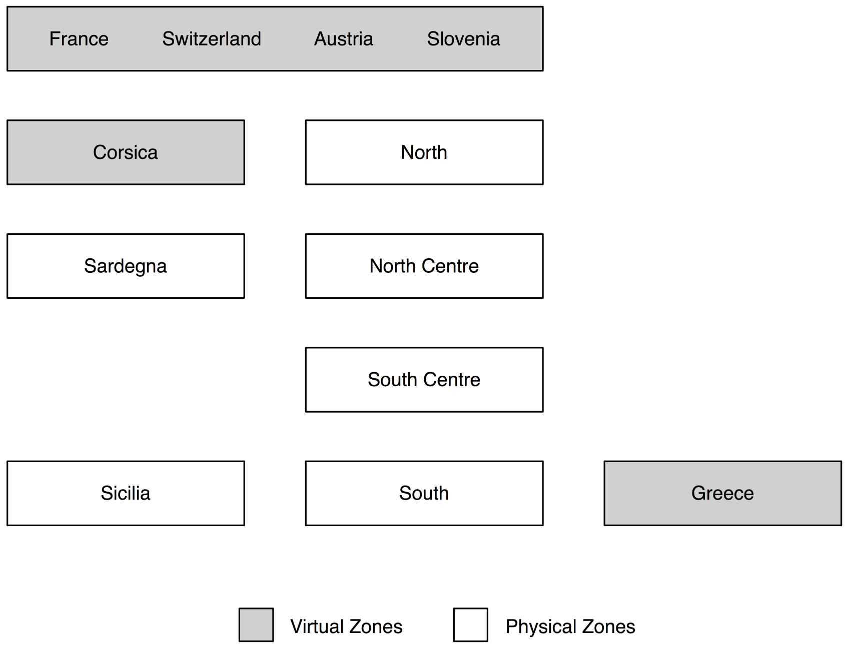


Figure 2: Italian Power System

Every geographical or virtual areas are a set of supply points; these points are the minimum electricity units to which must be defined the schedules of injection and withdrawal, whether defined in the execution of bilateral contracts or upon acceptance of bids/supply offers in the Electricity Market.

For each supply point is identified a "dispatching user". The user is responsible to Terna for carrying out the injections and withdrawal schedules, and balance them. Such orders can be sent by Terna to the supply points in real time to ensure both the safety of the system and the payment of expenses for the imbalance, that means penalties attributed to the supply points for lack of adherence of the schedules.

2.2 The Electricity Market

The Electricity Market is organized in:

- *Spot Electricity Market* (MPE),
- *Forward Electricity Market* with delivery obligation (MTE),
- *Platform for physical delivery* of financial contracts concluded on IDEX (CDE).

MPE is divided into two markets:

- *Day-Ahead Market* (MGP) where electricity is exchanged according to offers/demands and takes place in a single session in the implicit auction on the following day,
- *The Intraday Market* (MI) deals with the volume variations of electricity than which were traded in the MGP (which takes place into two implicit auctions with a different closing time).

The *Ancillary Services Market* (MSD) is also included in the Spot Electricity Market and it is divided into *ex ante MSD* and *Market Balance* (MB) (its role is to ensure the proper functioning of the dispatching service).

Operators participate in the market by submitting bids or supply offers that are make up of pairs of volume and its unit price (MWh; €/MWh) and express the willingness to sell (or buy) an amount of energy not higher than stated than the one of the bid/offer, at a price not lower (or not higher) than the specified one of the bid/offer.

Price and volume must not be negative and bids may not specify any purchasing price (except for MSD), this situation expresses the operator availability to purchase power at any price. Offers are referred to the "supply points" hour by hour (for each day and for each point of supply, may be submitted up to 24 bids and each is independent from the others).

Day-Ahead Market (MGP) hosts most of the transactions of electricity; this market is organized according to a model of implicit auction where hourly blocks of electricity are negotiated for the next day and where are defined, not only prices and quantities exchanged, but also schedules for injection and withdrawal for the next day.

During the sitting of MGP, operators may submit bids/offer shall indicate the amount and the maximum (minimum) at which they are willing to buy (sell). Each offer or bid must be made consistent with the potential injection or withdrawal of the supply point to which the bid/offer is related and must correspond to the will to inject or withdraw the electricity offered/asked.

The Supply Offers express the willingness to sell an amount of energy that does not exceed what specified in the offer at a price not less than that indicated in the offer itself. If this is accepted, leads to the commitment to inject the asked volume in the grid at a given time range.

The Demand Bids convey the will to purchase volumes of electricity not greater than what specified in the bid and at a price not higher than the one stated in the bid.

For such offers/bids operators can refer only to offer points for withdrawal or mixed.

Offers and bids are accepted after the closing of the trading day on the basis of economic merit and within the limits of transit between areas.

All accepted offers and bids referred to mixed points or withdrawal points belonging to the virtual zones, are valued at the equilibrium price of the area to which they belong. The price is determined for each hour, the intersection of the demand curve and supply and differs from zone to zone in the presence of saturated transit limits.

Accepted demand bids referred to withdrawal points belonging to the geographical areas are valued at National Single Price (PUN), equal to the average of the zonal prices weighted for zonal consumption.

Before the sitting of the MGP, GME makes available, to the operators, information regarding: the expected energy demand for each hour and each zone and the maximum limits allowed for the transitions between neighboring zones for each hour and for each pair of zones.

At the end of the sitting, GME initiates the resolution process: for each hour of the day following, the algorithm of the market accepts the bids/offers to maximize the transition between zones.

The approval process takes into account all offers to sell, valid and reasonable received, and sorts them by price in an ascending aggregate supply curve; the same happens for the demand bids received that are arranged in decreasing order in an aggregate demand curve. The intersection of the two curves determines: the total quantity traded, the equilibrium price, the accepted bids and schedules of injection and withdrawal obtained as sum of accepted bids/offers refer to the same area and the same supply point.

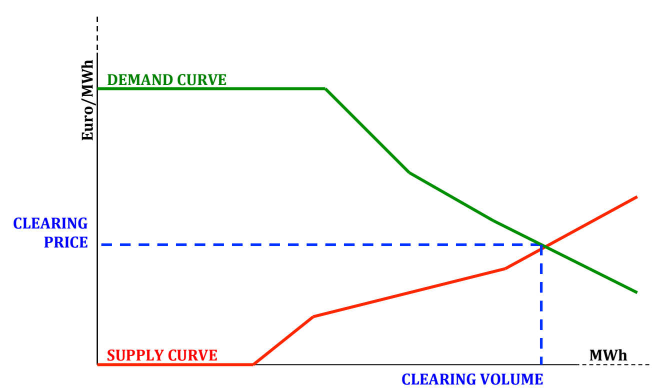


Figure 3: Determination of Clearing Price

If the flows on the grid resulting from the schedules do not violate any transmission limit, the equilibrium price is unique in all zones (and equal to the *clearing price* of Figure 3). The accepted bids/offers shall be those with a selling price (P*):

$$selling\,price \leq P*$$
$$purchasing\,price \geq P*$$

If at least one limit is violated, the algorithm separates the market in two market zones: one export zone that includes all the zones upstream, and one downstream for what regards the importation.

For each market zone is generated a supply curve (which includes all the offers presented in the same zone as well as the maximum volumes imported) and a demand curve (including all the bids presented in the area, as well as the volumes equal to the maximum quantity exported).

The outcome is an equilibrium price zone (Pz) in the two different market zones:

$$Pz_{exporting\,market\,zone} > Pz_{importing\,market\,zone}$$

This market separation process is repeated until the result is compatible with the grid constrains.

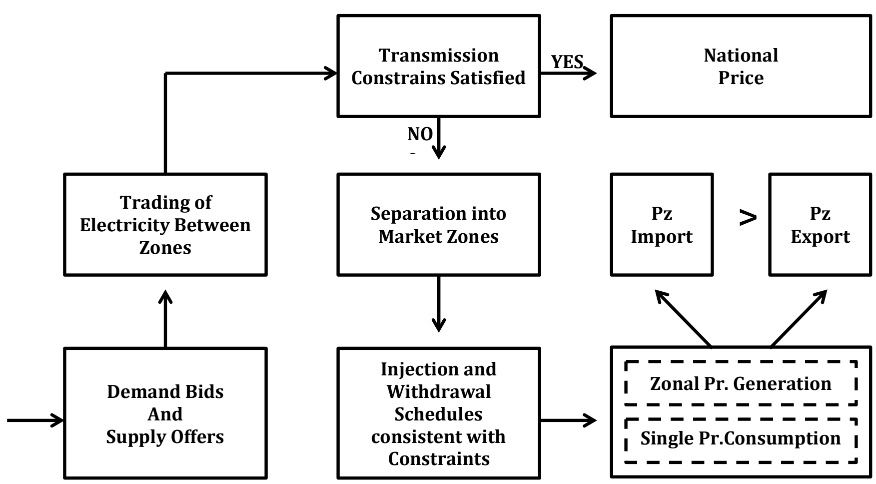


Figure 4: Determination of Clearing Price

3 Data

Electricity Prices

The dependent variable of the model is constituted by six vectors, each one refers to a macro energetic region. Each series consists of 35,060 zonal prices, from the 1/1/2008 to 31/12/2011, recorded by GME s.p.a. during the bargaining sittings.

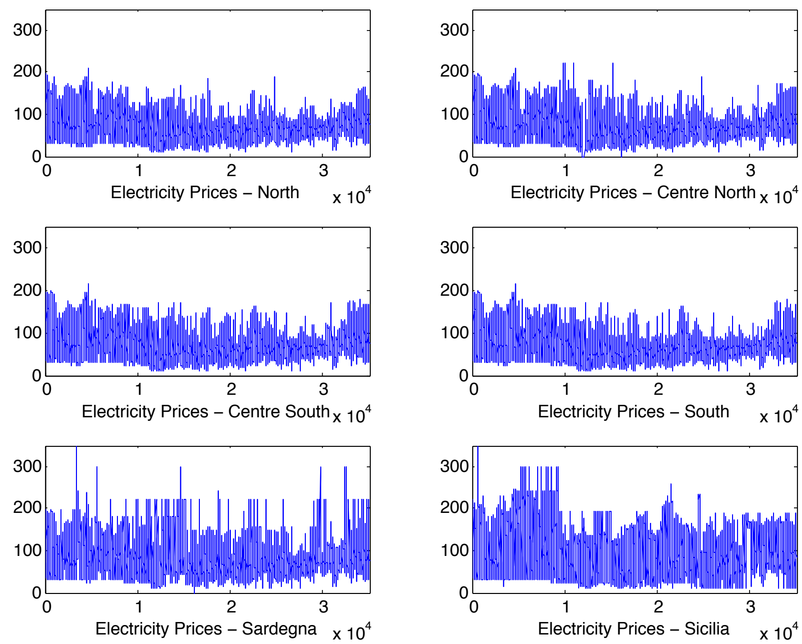


Figure 5: Hourly Electricity Prices - 2008 2011

Running some simple tests and looking at the histogram reported in the figure below, I can assume that the distributions of the zonal prices do not follow a normal distributions. This assumption is also testified by the Jacque-Bera test which rejects the null hypothesis of normality in every zone.

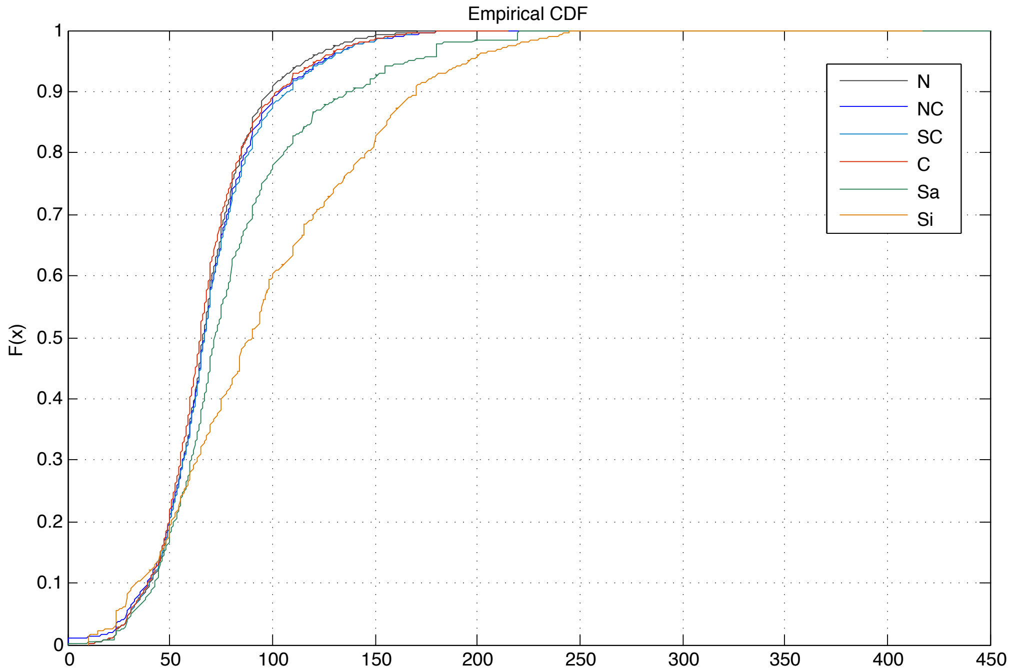


Figure 6: *Cumulative Density Function* of Zonal Prices

The shape of the distributions is pretty the same for each area, even if the observations of the peninsular zones are more centered around the mean than Sardegna and Sicilia (higher Standard Deviation). This could be a signal of the presence of higher costs for the "italian islands".

Electricity has to be considered as a singular commodity because, unlike the others, it cannot be stored: generation and consumption have to be constantly balanced in real time. This particular feature has economic consequences on the electricity price itself as shown in Table (1).

	North	**North C**	**South C**	**South**	**Sardegna**	**Sicilia**
Mean	68.99	69.998	70.88	68.74	81.82	97.65
Median	66.20	66.95	67	65	72	90
Max	209.98	220.50	215	215	300	417
St. Dev.	25.153	28.277	27.735	26.892	39.75	52.65
Kurtosis	5.1876	5.255	4.8976	5.44	5.32	3.28
Skewness	0.96	0.9332	1.0313	1.1705	1.3834	0.697

Table 1: Data Statistics (2008 - 2011) - Electricity Prices

Descriptive analysis highlights what has already appeared from the graph: mean and median are not close to each other; positive skewness indicates that the tail on the right side is longer than the left side one and the bulk of the datas lies to the left of the mean.

```
%ADF Test _ Results (North)

% results=adf(Y(:,1),0,24)
results =
     meth: 'adf'                crit: -3.4583
     nobs: 35033                      -2.8710
     nvar: 26                         -2.5937
     sige: 74.1840                    -0.4516
     rsqr: 0.8829                     -0.1060
     rbar: 0.8828                      0.5367
       dw: 2.0348
     nlag: 24
    alpha: 0.9595
      adf: -12.2621
```

The Dickey-Fuller test leads to the conclusion of rejection of the null hypothesis: no presence of unit root on levels (with 24 lags).

Volumes

Offered and demanded volumes are two of the exogenous variables taken into account in the analysis. From the graphs below emerges that the two variables have almost the same pattern in each area.

The difference among energetic regions is given by the amount of energy used: *North* has a electricity demand that is almost three times higher than the one of other regions. The geographic location (it is close to the neighboring countries) and the massive presence of industries in northern Italy could be two possibly reasons for this discrepancy also considering that the lowest electricity demands come from highly rural areas such as the south or the islands.

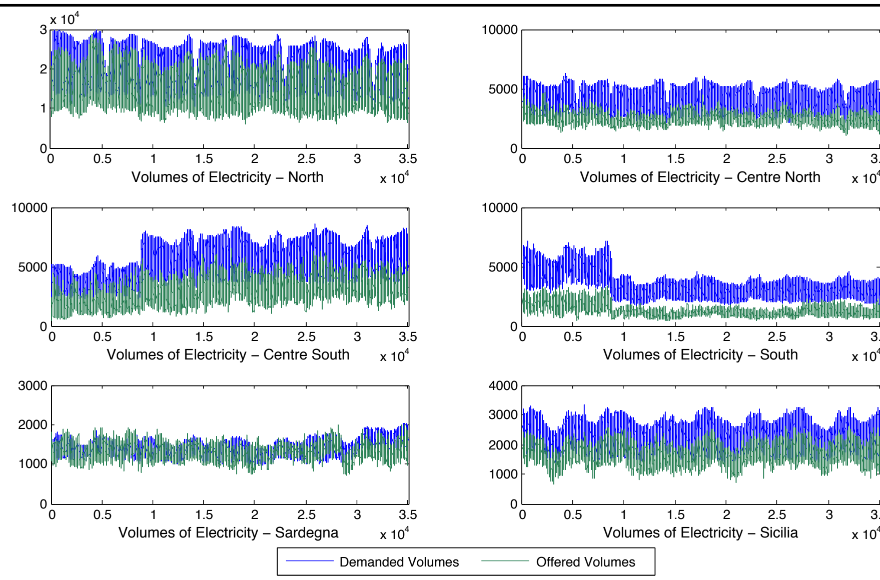


Figure 7: Zonal Demanded and Offered Volumes (2008 - 2011)

Another hint, given by the graphs, is about the zonal electricity production. In the first graph (North) the difference between offered and demanded volumes could be considered as the zonal actual consume, so the most of electrify is sold to the other regions with fewer productive structures. This statement is confirmed by the presence of 1613 facilities in the North of Italy, 277 in the Centre and 172 in the South and in the islands[1].

Temperatures

Variations of daytime/nighttime consumption, weather conditions and succession of seasons are all factors that affect the price and the demand of electricity.

Among the explanatory variables, I chose the temperatures, as proxy of all these factors.

[1]Data refer to hydroelectric power stations - www.gazzettadisondrio.it – 20 V 07 – n. 14/2007, anno IX°

My analysis includes 18 italian provinces selected as representative of the weather of each energetic zone. Every single macro region has at least one province whose data is used to describe the climate of the area:

- **North** (Bologna, Brescia, Genova, Milano, Rimini, Torino, Trieste, Venezia)[2],
- **Central North** (Firenze, Perugia)[3],
- **Central South** (Roma, Napoli, Pescara)[4],
- **South** (Bari, Reggio Calabria)[5],
- **Sardegna** (Cagliari)[6],
- **Sicilia** (Palermo, Catania)[7].

I retrieved all the information I needed through the use of a computer script that allowed me to get hourly frequency temperatures, otherwise difficult to categorize.

Presence of missing data is a common feature of these historical series. Each weather station, responsible for data collection, gathers the temperature information in its own way; this means that, considering a given sample period, some station specific temperature series could exhibit missing values when other temperature do not.

In order to fill the gaps in the time series of the weather data I followed two different approaches. On the one hand when there were just few isolated missing datas, I calculated the average between the previous and next value:

$$V_{missing,t} = \frac{(V_{t-1} + V_{t+1})}{2} \ ,$$

on the other hand, where the missing data were more than one in a row, I had to estimate the values by a multiple regression analysis.

Missing data were estimated as:

$$V_{0,t} = a_0 + \sum_{i=1}^{n} (a_i V_{i,t}) + \varepsilon_t \tag{1}$$

where

- $V_{0,t}$ is the missing data,

[2] Shown below the weather station codes as downloaded from the web site *www.wunderground.com*: LIBP, LICC, LIMF, LIMJ, LIRA, LIRQ, LIRZ, LIVT.

[3] LIEE, LIPL.

[4] LIML, LIPR, LIRN.

[5] LIBD LIPZ

[6] LICJ.

[7] LICR, LIPE.

- a_0, ..., a_n are the regression coefficients,
- $V_{i,t}$ is the value if the i^{th} weather station.

For instance, if we consider a presence of *"N/A"* observations between 8am and 12am in the *LIVT series*; to fill the blanks I used the information contained in neighboring provincial vectors (*LIRA, LIRZ, ...*), from the same time range, to run the regression (1).

3.1 Spectral Analysis

This method allows to understand the regular behavior of a time series and reveals in some way what we could expect from the market (see C. W. J. Granger(1969), "Investigating causal relations by econometric models and cross-spectral methods," Econometrica, vol. 37, no. 3). S*pectral analysis* lets us discover the seasonalities of a time series, looking at the content of the frequency distribution (the *spectrum*).

As the frequency is the inverse of the period of a signal, once obtained the frequency spectrum, we identify also the periodic components (cycles) of which it is composed, the signal and the strength of the cyclicities included in the series.

The analysis of time series in the frequency domain allow us to select the most relevant frequencies and to estimate the strength of the periodic component at a given frequency.

Fourier Analysis[8]

To analyze discrete time series (for example, a time series of prices) is generally used the Fourier approach; it allows to identify frequencies explaining a portion of seasonal variations in electricity prices.

The techniques of the Fourier analysis allow modelling a time series with seasonal components as a sum of periodic sinusoidal functions $A \cdot sin(\lambda t + \varphi)$, where A denotes the *amplitude* of a sinusoidal wave, λ the *frequency*, and φ the *phase shift*.

Assuming that λ is known (determined by the Fourier transform), estimates of the slope parameters can then allow calculating the respective amplitude and phase shift.

The Fourier transform of a real-valued function $p(t)$ on the domain $[0, T]$ is defined:

$$F(i\lambda) = F\left\{p(t)\right\} = \int_0^T p(t)e^{-i\lambda t}\, dt$$

where i is the imaginary unit such that $i^2 = -1$.

Based on this definition, the FFT numerical procedure computes

$$F(i\lambda k) \approx \sum_{t=0}^{T-1} p_t\, e^{-i\lambda_k t}$$

It is important to note that the values of the Fourier transform are complex numbers and are therefore not directly comparable, to avoid the problem is frequently used

[8]The results have been computed using the FFT procedure implemented in MatLab. See the Appendix at the end of the thesis for the program's lines.

the modulus of the Fourier transform. Figure 8 presents a *spectral densities* for hourly electricity prices.

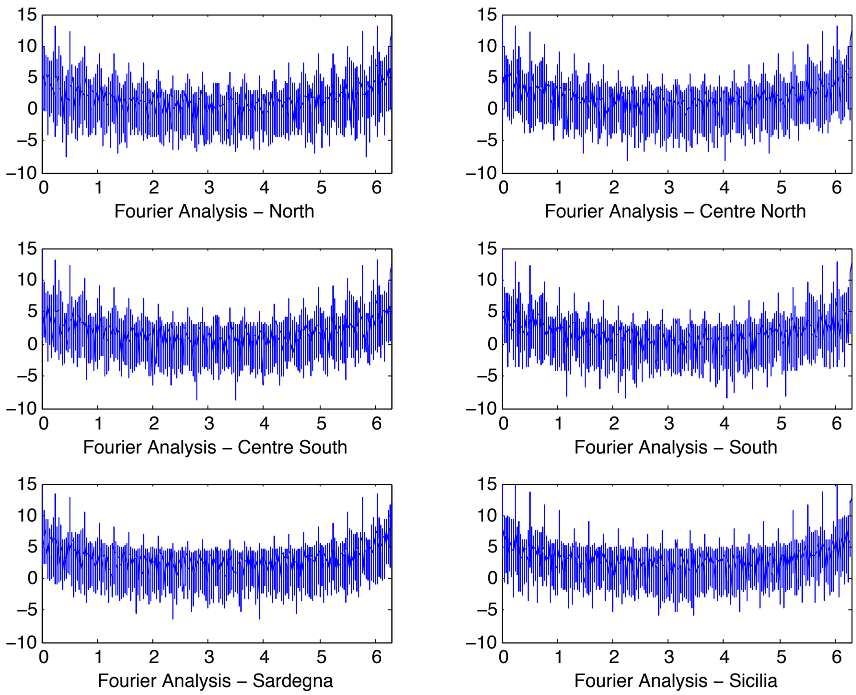


Figure 8: Fourier Analysis of Electricity Prices

The non storability of electricity is the main cause of seasonality presence in the dataset. The use of electricity has to be instantaneous, so the higher use of it during the daytime cannot be supply by an electricity generation during the night-time. This feature determines the occurrence of a daily seasonality. Other kinds of seasonalities observed in the frequency are the weekly and the annual ones. Weekly seasonality takes into account the difference between the weekends and the working days (where the demand is affected by industrial production). The annual seasonality is related to the climatic conditions.

In the following table are highlighted the peaks calculated by using the Fourier approach.

To obtain the hourly values of the seasonalities, 2π were divided by each value reported in the highest part of the table (Lambda). From the results emerges two different kind of seasonalities: the weekly one (described before) and the daily one[9].

[9]To get the Frequency: $\frac{\lambda}{2\pi}$ - To get the hourly cycles of ciclycities: $\frac{1}{Frequency}$

We could notice that the peaks describe more than the 2 seasonalities, for example we register also a 3-hours, a 6-hours and a 8-hours cycles. I will not include them in the model, because they may be considered as glares of the 24-hours seasonalities (also called harmonics).

Peaks in the Spectral Density - Fourier Analysis of Seasonalities (*Lambda*)

North	3.142	2.88	2.618	2.356	2.093	1.833	1.571	1.309	1.047	0.7857	0.5238	0.262	0.000179
North Central	3.142	2.88	2.616	2.356	2.093	1.834	1.571	1.309	1.047	0.7857	0.5238	0.262	0.000179
South Central	3.142	2.879	2.618	2.356	2.093	1.833	1.571	1.309	1.047	0.7857	0.5238	0.262	0.000179
South	3.141	2.88	2.618	2.355	2.093	1.832	1.57	1.309	1.047	0.7857	0.5238	0.262	0.000179
Sardegna	3.141	2.882	2.618	2.354	2.093	1.832	1.57	1.309	1.047	0.7857	0.5238	0.262	0.000179
Sicilia	3.142	2.88	2.618	2.357	2.095	1.833	1.571	1.309	1.047	0.7857	0.5238	0.262	0.000179

Fourier Analysis of Seasonalities (*Frequency*)

North	0.500	0.458366	0.417	0.37497	0.33	0.2917	0.25	0.208	0.166	0.125048	0.08336536	0.041698	2.852E-05
North Central	0.500	0.458366	0.41635	0.37497	0.33	0.29189	0.25	0.208	0.166	0.125048	0.08336536	0.041698	2.852E-05
South Central	0.500	0.458207	0.417	0.37497	0.33	0.2917	0.25	0.208	0.166	0.125048	0.08336536	0.041698	2.852E-05
South	0.4999	0.458366	0.417	0.3748	0.33	0.29157	0.249	0.208	0.166	0.125048	0.08336536	0.041698	2.852E-05
Sardegna	0.4999	0.458684	0.417	0.37465	0.33	0.29157	0.249	0.208	0.166	0.125048	0.08336536	0.041698	2.852E-05
Sicilia	0.500	0.458366	0.417	0.37513	0.33	0.2917	0.25	0.208	0.166	0.125048	0.08336536	0.041698	2.852E-05

Fourier Analysis of Seasonalities (*Hours*)

North	1.999	2.181661	2.40	2.6669	3	3.4278	3.999	4.799	6	7.99692	11.99539	23.98162	35062.418
North Central	1.999	2.181661	2.40183	2.6669	3.002	3.4259	3.999	4.799	6	7.99692	11.99539	23.98162	35062.418
South Central	1.999	2.18242	2.40	2.6669	3.002	3.4278	3.999	4.799	6	7.99692	11.99539	23.98162	35062.418
South	2	2.181661	2.40	2.668	3.002	3.4297	4	4.799	6	7.99692	11.99539	23.98162	35062.418
Sardegna	2	2.180147	2.40	2.669	3.002	3.4297	4	4.799	6	7.99692	11.99539	23.98162	35062.418
Sicilia	1.999	2.181661	2.40	2.666	3	3.4278	3.999	4.799	6	7.99692	11.99539	23.98162	35062.418

Cycles Recorded by Fourier Analysis

2h	2h 10m	2h 24m	2h 40m	3h	3h 25m	4 h	4h 48m	6h	8h	12h	24h	4 year

Table 2: Find Peaks - Fourier Analysis of Cyclicities

4 The Bayesian Approach

Because of the regional division imposed by the Italian government to manage the wholesale market, and given the dataset chosen seeking to explain the relationship between the exogenous variables and the spot electricity prices, I decided to approach the analysis considering each electricity zone as “a country on his own”.

One of the most successful models, flexible enough to be applied, was the *Vector Autoregressive Model (VAR)* which has proven to be very useful for describing the dynamic behavior of economic and financial time series. Recourse to the VAR granted me to take into account lagged interdependencies (including the lagged spot electricity price series in the regression) and potentially also unit specific dynamics. These features generated a very large number of coefficients that did not admit to use a classical estimation method.

To solve the issue,it comes in handy the Bayesian approach, with which I could restrict the coefficient vector to depend on a low dimensional vector of time varying factors. These factors may capture variations in the coefficients that are common across zones and variables (have a common effect). The variables considered could be included in the prior assumptions as *unit-specific* (they affect just one or few ares) or they may be related to all the regions.

My analysis will start taking into account all the endogenous variables as if they were related with all the geographic ares considered. This will be possible using a *non informative prior* and a *natural conjugate* one. Later I will impose more restrictive assumptions on parameters, considering temperatures and volumes as unit specific, and inserting these expectations in the prior formulation.

To define the fundamental characteristics of the Bayesian method, we could consider y as a data vector (or matrix) and θ as a parameters vector (or matrix) that seeks to explain y. Having established y and θ, by using the *Bayes Rule*[10], the goal of the researcher is answer the question: “Given the data, what do we know about θ?”(Gary Koop, *Bayesian Econometrics* 2003).

So what is important in the approach, is learning about θ given “something known” as the data (y), or better, is being interested in using the data to understand the “role” of parameters in the model. This is allowed by the next relationship derived from the Bayes rule:

$$p(\theta|y) \propto p(y|\theta)p(\theta)$$

where the term $p(\theta|y)$ is referred to the *posterior density*, $p(\theta)$ is the *prior density*

[10] Let consider: $p(A,B) = p(A|B)p(B)$ and $p(A,B) = p(B|A)p(B)$ *where A and B are two events of a given probability ;*

From the previous two equations we can obtain $p(B|A) = \frac{p(A|B)p(B)}{p(A)}$ which is known as Bayes Theorem for events.

and $p(y|\theta)$ is the *likelihood function*[11] which is the density of the data conditional on the parameters of the model.

So Bayes theorem transforms prior or initial probabilities, $p(\theta)$, into posterior or subsequent probabilities, $p(\theta|y)$ which combines data e non-data informations. Formulate an economic model means collect probability distributions conditional on different values for θ, about which a researcher wish to learn; then the beliefs of the researcher about θ must be organized into a (prior) probability distribution. After collecting the data and inserting them into the family of given distributions, using the Bayes' theorem, it is possible to calculate new beliefs about θ.

4.1 The Model

General Specification

Let start considering a generalized specification of the model used for the analysis:

Defined Y as a $(T \times M)$ matrix, X as a matrix which stacks the T observation of each regressor included in the model, and A as a matrix that contains all the coefficients including the intercept $A = (a_0\ A_1\ ...\ A_p)'$; given these premises the model could be written as:

$$Y = XA + \varepsilon \tag{2}$$

or, considering $\alpha = vec(A)$ as a $(KM \times 1)$ vector in which are included all the VAR coefficients (and the intercepts), the same model could be written as:

$$y = (I_M \otimes X)\,\alpha + \varepsilon \tag{3}$$

where $\varepsilon \sim N(0, \Sigma \otimes I_M)$.

Now, because we are interested in highlight which kind of regressors are included in X, then we can represent the model as:

$$y_{it} = \alpha_i + \sum_{j=1}^{P}\sum_{l=1}^{M}\phi_{j,il}\, y_{t-j,l} + x'_{it}\beta_i + \varepsilon_{it} \tag{4}$$

where:

we can notice the presence of the lagged term $y_{t-j,l}$, descriptive of the lagged y included;

x_{it} is the vector of the unit-specific regressor:

[11] See Section 5, *Appendix A*

$$x_{it} = \begin{bmatrix} x_{1t} \\ x_{2t} \\ \dots \\ x_{kt} \end{bmatrix}$$

and ε_{it} may be considered as sum of different effects:

$$\varepsilon_{it} = \psi_i + \delta_t + u_{it}$$

where its components could be defined as: a unit specific effect ψ_i , the time effect δ_t and the disturbance term u_{it} .

To simplify the formula (1) let drop the index j from $\phi_{j,il}$ by considering just 1 lag, so we obtain:

$$y_{it} = \alpha_i + \sum_{l=1}^{M} \phi_{jl}\, y_{t-1,l} + x'_{it}\beta_i + \varepsilon_{it} \tag{5}$$

Assuming that:

$$Y_t = (y_{it}, \dots, y_{Mt})'$$

$$\alpha = (\alpha_{1,} \dots, \alpha_M)'$$

$$\Phi = (\phi_{il})\ con\, i, l = 1, \dots, M$$

The model could be reduced to:

$$Y_t = \alpha + \Phi Y_{t-1} + X_t\beta + \varepsilon_t \tag{6}$$

The Model (Variables Specification)

Considering the 6 geographical zones, for each area I set up the following model:

$$\underset{m\times 1}{y_{it}} = \underset{m\times m}{A_i}\ \underset{m\times 1}{y_{it-1}} + \underset{m\times q}{C_i}\ \underset{q\times 1}{z_{it}} + \underset{m\times 1}{\varepsilon_{it}} \tag{7}$$

with $t = 1, ..., T$ and $i = 1, ..., N$ where $N = 6$ and $T = 35060$

The model could be reduced to a more compact formula considering all the coefficients included in the model as region specific:

$$Y_t = \alpha + \Phi_1 Y_{t-1} + DZ_t + \varepsilon_t \tag{8}$$

Y_t contains the spot electricity price,

Y_{t-1} — its t-th row is $y_{it-1} = [y'_{it-1}, ..., y'_{it-1}]$ with $k = mp$, descriptive of the lagged electricity prices included in the model (in the analysis I included just one lag for the prices);

Z_t — is a *(T × q)* matrix of explanatory variables; that means it includes all the variables previously described in section 2: *hourly temperatures* the of the 18 italian provinces, *demanded volumes* of the six ares, *offered volumes*, *six weekly dummies* and *23 daily ones.*

We also assume that the distribution for ε is given by $\varepsilon \sim N(0, \Sigma \otimes I_M)$.

Considering $X_t = (I_n \otimes x_{t-1})$, $x_{t-1} = (Y'_{t-1}, ..., Y'_{t-p}, z'_t)'$ and $\beta = vec(\Phi_1, D)$ the model could be written as:

$$Y_t = X_t\beta + \varepsilon_t$$

In this way the unknown parameters of the model become β and Σ. The errors in each equation are homoskedastic and not autocorrelated. Using a probability density function (also known as *pdf*), the likelihood function for β and Σ can be written as:

$$L(Y|\beta, \Sigma) \propto |\Sigma|^{-T/2} exp \left\{ -\frac{1}{2} \sum_t (Y_t - X_t\beta)' \Sigma^{-1} (Y_t - X_t\beta) \right\} \tag{9}$$

Therefore, for a given prior $p(\beta, \Sigma)$, the posterior is going to be:

$$p(\beta, \Sigma|Y) \propto p(\beta, \Sigma) L(Y|\beta, \Sigma) \tag{10}$$

4.2 About Priors

General consideration about priors

A fundamental choice to implement the Bayesian estimation is the selection of a prior distribution for the parameters of the model. The Bayesian literature take into account specific proposal for different economic problems which means different ways to determine the parameters β and Σ of a prior. Use a "genuine" approach to the estimation would require to determinate a prior distribution also for the parameters of the prior $p(\beta, \Sigma)$ and then integrating them out of the posterior distribution; this is called *Full Bayesian Approach*. However, sometimes it may result difficult to implement this integration so it remains to find out some alternatives. One of them is substitute directly into the formulas for the mean and the variance of the posterior distributions, the estimation of hyper-parameters (Matteo Ciccarelli and Alessandro Rebucci (2003), *Bayesian VARs: A survey of the recent literature with an application to the European Monetary System*); estimation that could be OLS. This kind of solution to the problem is also called Bayesian Empirical Estimation.

In the development of the model for the Italian electricity prices I chose the Non Informative Prior and the Natural Conjugate one; anyway I want to give an overview of other priors that could suit with the model.

Minnesota Prior[12]

The Minnesota prior, Minnesota (MN) prior, first introduced by Litterman (1980), on the VAR coefficients is centered on the assumption that each variable follows a random walk process.

For Litterman the problem is estimating the $(k \times 1)$ vector β_g that contains the parameters of the g^{th} equation of $Y_t = X_t\beta + \varepsilon_t$ where the error term is known and equal to $\sigma^2_{g,g}$.

More precisely, this prior states that:

$$p(\beta_g) \sim N(\bar{\beta}_g, \bar{\Omega}_g) \tag{11}$$

where$\bar{\beta}_g$ and $\bar{\Omega}_g$ represents the prior mean and the variance-covariance matrix of β_g. The residual variance-covariance matrix, Σ, is assumed fixed and diagonal, $\sigma^2_{g,g} I_T$.

Vectorizing the time observation of the g-*th* equation we will obtain:

$$Y_g = X\beta_g + \varepsilon_g \quad , \quad g \equiv 1, ..., n \tag{12}$$

where Y_g and ε_g are $(T \times 1)$ vectors. So, given (12) and assumed the independence of the error terms: the likelihood function described at (9) becomes:

$$L(Y|\beta, \Sigma) \propto |\sigma^2_{g,g}|^{-T/2} exp \left\{ -\frac{1}{2\sigma^2_{g,g}} \sum_t (Y_g - X\beta_g)' (Y_g - X\beta_g) \right\} \tag{13}$$

Therefore, for the given prior and likelihood, the posterior could be obtain:

$$p(\beta_{\mathbf{g}}|Y) \propto |\sigma^2_{g,g}|^{-T/2} |\bar{\Omega}_g|^{-T/2}$$

$$exp \left\{ -\frac{1}{2} \left[\left(\beta_g - \bar{\beta}_g\right)' \bar{\Omega}_g^{-1} \left(\beta_g - \bar{\beta}_g\right) + \frac{1}{\sigma^2_{g,g}} \sum_t (Y_g - X\beta_g)' (Y_g - X\beta_g) \right] \right\}$$

[12] proposed by R.Litterman (1986), University of Minnesota

our posterior relation, considering that $|\sigma_{g,g}^2|^{-T/2}$ and $|\bar{\Omega}_g|^{-T/2}$ are constants and simplifying the products inside the parenthesis by $Y_g'Y_g$ and $\bar{\beta}_g\tilde{\Omega}^{-1}\bar{\beta}_g$ because constant too[13], the proportion will be:

$$p\left(\beta_{\mathbf{g}}|Y\right) \propto exp\left\{-\frac{1}{2}\left[\left(\beta_g-\bar{\beta}_g\right)'\tilde{\Omega}_g^{-1}\left(\beta_g-\bar{\beta}_g\right)\right]\right\} \tag{14}$$

with:

$$\beta_{\mathbf{g}} = \tilde{\Omega}_g\left(\bar{\Omega}_g^{-1}\bar{\beta}_g+\sigma_{g,g}^{-2}X'Y_g\right)$$
$$\tilde{\Omega}_g = \left(\bar{\Omega}_g^{-1}+\sigma_{g,g}^{-2}X'X\right)^{-1}$$

There are few consideration that is proper to remark: first of all the prior and posterior independence (without it is not possible to estimate them separately); Σ is assumed fixed and diagonal, with the diagonal elements obtained from an *AR(p)*; $\bar{\beta}_g$ and $\bar{\Omega}_g$ are unknown and specified in terms of few known hyper-parameters. Assuming a infinitive dispersion of the prior distribution around its mean (that could be obtain by the *Gibbs Sampler)* $\bar{\Omega}_g \to 0$, the posterior mean of $\beta_g$will become equal to $(X'X)^{-1}X'Y_g$, which is the OLS estimator of $\bar{\beta}_g$.

Litterman assuming that the most of the time series are well represented by random walk processes, decided to assign numerical values to hyper-parameters of the model. He considered Π as a *degenerate random variable* on the assigned values with a pre-determinated structure for the diagonal elements of the matrix $\bar{\Omega}_g$. Therefore, the variance of $\bar{\beta}_g$ is defined by:

$$\begin{cases} \pi_2/l^2 & for\ the\ g^{th}\ lag\ of\ endogenous\ variable \\ (\pi_3/l^2)\,\sigma_{g,g}/\sigma_{j,j} & for\ the\ g^{th}\ lag\ of\ endogenous\ variable\ (j \neq g) \\ \pi_4\,\sigma_{g,g} & for\ deterministic/exogenous\ variable \end{cases}$$

Given $l = 1, ..., p$ the number of lags of a variable, we could consider π_2 as the controller of the tightness its own lags, π_3 controls the tightness of the own lags relative to lags of the other variable in the equation and $\pi_4$controls the uncertainty on deterministic or exogenous variables while $\sigma_{g,g}$ and $\sigma_{j,j}$ measure the scale of fluctuation. Finally the mean vector is specified as $\bar{\beta}_g = (0, ..., 0, \pi_1, 0, ..., 0)$ where $\pi_1$is the prior mean of coefficient on first lag of endogenous variable in equation *g.*

[13]Matteo Ciccarelli and Alessandro Rebucci (2003), *Bayesian VARs: A survey of the recent literature with an application to the European Monetary System,* Research Department, International Monetary Fund (IMF) Working Paper

Non Informative Priors

Till now, we have not specified any prior information for the empirical model analysis, we just give an overview of the Minnesota Prior which is one of the most important prior assumptions of the Bayesian literature. By relaxing the previous hypothesis about the posterior independence between equations and the fixed residual variance-covariance matrix, we can introduce the next two priors, the ones we used for modelling the Italian electricity spot prices. Without a prior $p(\beta, \Sigma)$ is hard to obtain precise information for a model that involves many coefficients.

The conventional non informative prior is chosen to provide objectivity; it will especially become an useful tool in the comparison with the other priors:

$$p(\beta, \Sigma) = p(\beta)p(\Sigma) \propto |\Sigma|^{-(n+1)/2} \tag{15}$$

As said before, applying Bayes' rule to the prior pdf in (11) and the likelihood function in (9) yields the *joint posterior probability density function* for β and Σ:

$$p(\beta, \Sigma|Y) \propto p(\beta|\Sigma, Y)\, p(\Sigma|Y) \tag{16}$$

The conditional posterior of β given Σ and the data is distributed as a normal:

$$p(\beta|\Sigma, Y) \sim N\left(\hat{\beta}_{ols}, \Sigma \otimes (X'X)^{-1}\right) \tag{17}$$

with posterior mean equal to the generalized least squares estimator.

The conditional posterior of Σ given the data is distributed as an Invers Wishart distribution[14] with parameter matrix $(Y - X\hat{B}_{ols})'(Y - X\hat{B}_{ols})$, the sum of squared error of the OLS estimation, and degrees of freedom $T - k$:

$$p(\Sigma|Y) \sim iW_{T-k}\left[(Y - X\hat{B}_{ols})'(Y - X\hat{B}_{ols})\right] \tag{18}$$

with X of dimensions $(T \times k)$ and $Y$of dimensions $(T \times n)$ denoting the matrix version of X_t and Y_t, $B = [B_1', ..., B_p', D']$ of dimensions $(k \times n)$ and $iW(Q, q)$ denoting an inverted Wishart distribution with scale matrix Q and q degrees of freedom.

In order to obtain draws of β and Σ from their own marginal posterior probability density function, one possible way is to use an algorithm for sampling from probability distributions using the *Monte Carlo* approach based on the construction of a *Markov Chain*.

[14]See Appendix A

Markov Chain Monte Carlo (MCMC) is widely applicable to Bayesian problems and better known as *Gibbs sampler*. In this procedure draws are made iteratively from the conditional posterior pdf. Given a particular starting value for Σ extracted from the Inverted Wishart distribution, that may be called as $\Sigma_{(0)}$, the i-th draw from the Gibbs sampler $(\beta_{(0)}, \Sigma_{(0)})$ is obtained using the following two steps:

(a) draw $\beta_{(i)}$ from $p(\beta|\Sigma_{(i-1)}, Y)$,

(b) draw $\Sigma_{(i)}$ from $p(\Sigma|\beta_{(i)}, Y)$.

As already said, the two conditional posterior pdf's are normal and inverted Wishart, respectively.

After a sufficiently high number of draws, the Markov Chain created by the draws will converge to a determined beta and sigma. After this convergence, the next draws can be viewed as draws from the marginal posterior pdf's p(β|Y) and p(Σ|y). These draws can be used to obtain the results. Draws that take the prior to the point of convergence are discarded.

Assessing whether convergence has taken place is similar to assessing whether a time series is stationary (William E Griffiths, *Bayesian Inference in the Seemingly Unrelated Regressions Model* - April 18, 2001). Then we proceeded calculating the posterior mean and the posterior standard deviation of the "beta draws" and the "sigma draws" resulted of the Gibbs loop.

Integrating Σ out of the joint posterior distribution, the marginal posterior distribution of B (the matrix from the parameter vector of β), $p(B|Y)$ become:

$$p(B|Y) \propto \left|(Y - \hat{B}_{ols})'(Y - \hat{B}_{ols}) + (B - \hat{B}_{ols})'X'X(B - \hat{B}_{ols})\right|^{-T/2}$$

which is a generalized t-Student distribution with scales$(Y - \hat{B}_{ols})'(Y - \hat{B}_{ols})$ and X'X, mean $\hat{B}_{ols}$ and degrees of freedom $T - k$.

Natural Conjugate Prior

The second class of prior we used to interpret and make computation is the *Natural Conjugate Prior*. We proceed with a *conjugate family* of prior distributions because it is a convenient way of solving the main drawbacks of the Minnesota prior.

Conjugancy is the property by which the posterior distributions follows the same parametric form of the prior distribution; it is defined considering: $\Im$ as a class of sampling distributions $p(y|\theta)$, $\aleph$ as a class of prior distributions for θ, then the class $\aleph$ is a *conjugate* for $\Im$ if $p(y|\theta) \in \aleph$ for all the $p(\cdot|\theta) \in \Im$ and $p(\cdot) \in \aleph$.

Natural conjugate priors arise by taking $\aleph$ to be the set of densities having the same functional form as the likelihood (German et al. 1995).

Priors are meant to reflect any researchers' informations, before seeing the data that wish to be included. When we combine a conjugate prior distribution with the likelihood, it yields a posterior that falls in the same class of distributions for β and Σ.

One of its advantage is that the *natural conjugate* has the same functional form of the likelihood function, this means that the output information can be interpreted in the same way as likelihood function information or, broadly speaking, the prior can be interpreted as coming from a hypothetical sample of data set from the same process that generated the data.

Let consider $\bar{\beta}$, the vectorization of a matrix $M \times k$ (66×6, in the analysis), as the prior mean of beta; Σ as a single draw resulted from the sigma posterior and $\bar{\Omega}$ as the prior variance of beta.

To relax the assumption of a fixed and diagonal variance-covariance matrix of residuals, the natural conjugate prior for normal data is the Normal-Wishart[15]:

$$p(\beta|\Sigma) \sim N(\bar{\beta}, \Sigma \otimes \bar{\Omega}) \tag{19}$$

$$p(\Sigma) \sim iW(\bar{\Sigma}, \alpha) \tag{20}$$

with $\bar{\Sigma}$ as prior scale of sigma and α its degree of freedom (7 in the computation). Briefly, $\bar{\beta}$ is specified as dependent upon only one hyper-parameter that controls the mean of the first lag of the endogenous variable, $\bar{\Omega}$ is specified as a diagonal matrix, and the diagonal elements of $\bar{\Sigma}$ are estimated from univariate *AR(p)* models ($p = 1$ in the model); α, prior degrees of freedom, has to be chosen to ensure the existence of the prior variances of parameters. Given the prior assumptions, the posterior distributions is obtain as:

$$p\left(\beta|\Sigma, Y\right) \sim N(\tilde{\beta}, \Sigma \otimes \tilde{\Omega}) \tag{21}$$

$$p(\Sigma|Y) \sim iW(\tilde{\Sigma}, T + \alpha) \tag{22}$$

where:

$$\tilde{\Omega} = \left(\bar{\Omega}^{-1} + X'X\right)^{-1}$$

$$\tilde{B} = \tilde{\Omega}\left(\bar{\Omega}^{-1}\bar{B} + X'X\hat{B_{ols}}\right)$$

$\tilde{\Sigma}$ given the data and $T + \alpha$ degrees of freedom (35066) is equal to:

[15]Unconditional prior distribution of β will be normal with prior mean $E(\beta) = \bar{\beta}$ and variance $V(\beta) = (a - n - 1)^{-1}\bar{\Sigma} \otimes \bar{\Omega}$ where α denotes the degree of freedom of the inverse-Wishart and satisfied $\alpha > n + 1$

$$\tilde{\Sigma} = \hat{B_{ols}}' X' X \hat{B_{ols}} + \bar{B}' \bar{\Omega}^{-1} \bar{B} + \bar{\Sigma} + (Y - X\hat{B_{ols}})'(Y - X\hat{B}_{ols}) + \tilde{B}' \left(\bar{\Omega}^{-1} X' X\right) \tilde{B}$$

as in the previous prior situation (Non Informative Prior), integrating Σ out of the joint posterior, the marginal posterior distribution of B is a multivariate t-Student distribution, whose integration can be performed numerically (Kadiya and Karlsson,1997).

Working with conjugate priors means assume that $\bar{\beta}, \bar{\Omega}$ and $\bar{\Sigma}$ are known otherwise should be adopted a Minnesota-type specification for these matrices.

"*Strict*" Prior

In the model evaluation we used a Non Informative and a Normal Wishart Conjugate priors settings. We consider the effect of all the variables included in the model as not unit-specific. So, as we could see in the next section, also the temperature of a southern province could be significative in defining the electricity spot prices of the northern region.

To improve the estimate results we could use the prior beliefs that not all the explanatory variables are essentials for price of a given region. We proceed imposing more strict restrictions on the prior distribution of the parameters β and Σ.

The constrains consist in imposing zeros for the parameter I do not assume related with all the Italian regions. The new imposed conditions can be summarized as follows: the *lagged spot prices* will continue to be related with each geographical area, the *temperatures* will be divided according to zone from where they belong (i.e.: The 8 temperature of the north will be related, in prior assumptions, just with the north), the *volumes* (demanded and offered) will follow the same path of the temperature so they will be considered as region specific and finally, the *dummies* (both weekly or daily) will continue to be referred to all the regions.

5 Empirical Results

In this section we want to give an overview of the results for the three different prior choices (described in section 4) for our BVAR Model.

In *Appendix B,* I reported the completed version of graph and tables of the posterior beta mean arising from the 500 draws created by the Gibbs sampler. In the next paragraph, after describing the output of the Non Informative prior (the weakly restrictive), we will proceed using its results to made a comparison with the other two priors. First, the Normal Conjugate and than the Strict Prior.

The significance of the coefficients is graphically represented in the tables by the colored cells and obtained using the credibility intervals given by the quantiles 0,05 and 0,95. If zero belongs to the interval described by the quantiles, it is high credible that the posterior mean (beta mean) is equal to zero (white cell), if zero does not belong to the credibility interval than it is highly credible that the posterior mean is not equal to zero (cells light gray or gray).

Non Informative Prior

Observing the reported results in Table (9), we can draw several conclusions.

As we could have expected the Lagged Prices (with $p = 1$) play a relevant role in capturing the variations of hourly spot prices, especially the ones related with their own country, in fact all these coefficients are positive, high significative and their values are included between 0.64 and 0.85.

Sicilia lagged spot prices is the only variable that does not affect none of the others, followed by Sardegna Lagged Prices where, even if its coefficients are significant their impact on the spot prices of *North Centre* and *South* to which they refer, is negligible.

Something interesting appears in the relation between the coefficients of the lagged prices of South and South Centre with the North. It shows that, even if they are significative and captures part of the variations of spot prices, their effect is counterbalanced[16].

[16] See also Table 1, where are reported some statistics about the distribution of South Prices and South Centre prices.

	N	NC	SC	S	Sa	Si
Bologna	0.135426	0.140840	0.140431	0.136542	0.102341	0.251934
Brescia	-0.004505	0.040576	-0.000696	0.002509	0.122437	-0.048769
Genova	0.057909	0.062450	0.050846	0.064274	-0.013470	-0.075861
Milano	0.146650	0.184483	0.174758	0.172834	0.058240	0.199955
Rimini	0.020528	0.034278	0.031859	0.003263	0.016034	-0.120713
Torino	-0.078177	-0.053990	-0.066346	-0.087594	0.076559	0.025430
Trieste	-0.061890	-0.087163	-0.081372	-0.062508	-0.115006	-0.159496
Venezia	-0.025992	-0.043725	-0.058470	-0.072602	-0.000522	-0.023310
Firenze	0.068910	0.044952	0.063828	0.056073	0.072906	0.098447
Perugia	0.017751	-0.016958	0.023071	0.015189	0.183209	0.107897
Roma	-0.135716	-0.140327	-0.158270	-0.163440	-0.096174	-0.233462
Napoli	0.180555	0.195615	0.188458	0.181355	0.153701	0.257061
Pescara	0.053319	0.054762	0.052254	0.041134	0.027940	0.254655
Bari	-0.147389	-0.178772	-0.113518	-0.118499	-0.181744	0.025355
Reg_Cal	-0.012079	0.000196	0.032317	0.068868	0.027916	0.195999
Cagliari	0.041097	0.072879	0.021279	0.046643	-0.049963	-0.242790
Palermo	-0.200879	-0.265178	-0.226696	-0.216144	-0.150067	-0.239205
Catania	-0.074174	-0.062755	-0.069663	-0.059405	-0.133372	-0.206837

Show coefficients with a credibility interval which does not contain zero

Show coefficients with a credibility interval which contain zero

Table 4: *Non Informative Prior* - Posterior Mean relations of Temperature and Zones

From the observation of the Temperatures' coefficients (Table 4), using the Non Informative prior, the most of the coefficients result significative and, in few cases, the are relevant in capturing the variation of SPs[17]. For instance, a temperature increase in Palermo or Catania seems to generate a decrease in the spot prices of peninsular regions. The presence of negative coefficients is a common feature of the provinces belonging to the "middle" or southern Italy (with the exception of Napoli that shows positive and significative coefficients related to all the regions $0.15 \leq \beta_{Temp_{Naples}} \leq 0.25$). The opposite situation is highlight by the northern temperature whose influence on prices leads to a proportional increase of them at the rising of the temperatures.

Volumes represent the demand and the offer of electricity for each region; what emerges is that an increase in electricity demand by Sicilia (or by the North in relation to SC[18] and Sa[19]) is linked to a statistically significant decrease in the price of other regions (except for

[17]Spot Prices
[18]SC: $\beta_{D\_Vol_{North}} = -0.000224$; Sa: $\beta_{D\_Vol_{North}} = -0.000746$;
[19]SC: South Centre; Sa: Sardegna

Sardegna)[20]. The same happens concerning the offered loads side for the South Centre, where the reduction in its offered volumes generates a proportional increase in the prices of other regions (except for Sardegna)[21].

As for the lagged prices, the offered volumes of Sicilia turn out to be not significant for all the zones and an analogous situation is shown by the OV[22] coefficients of Sardegna.

We included in the model weekly and hourly dummies in order to capture the intra day and weekly seasonalities. All the hourly dummies are significative positive (except D_2, D_3, D_4, D_{22} which are non significative for some ares), just the opposite of the weekly dummies that are all significantly negative.

What emerges is that the electricity prices tend to increase from the early morning to afternoon then they decrease till midnight describing and capturing variations in the daily use of electricity. On the other hand weekly dummies highlight the cyclical pattern in spot prices due to the variations in business or residential use. Besides that, as reflected in the dummies, the spot prices results lower during the weekend than in the working days.

[20] $-0.0034 \leq \beta_{D\_Vol_{Soutu\,Centre}} \leq -0.001$
[21] $-0.004 \leq \beta_{O\_Vol_{Soutu\,Centre}} \leq -0.0036$
[22] offered volumes

Natural Conjugate Prior

	N	NC	SC	S	Sa	Si
L_N	0.786656	0.014667	0.051262	0.055907	0.047668	0.044037
L_NC	-0.006321	0.839506	-0.023132	-0.022474	-0.052735	-0.044613
L_SC	-0.095805	-0.169758	0.646372	-0.093279	-0.078755	-0.063825
L_S	0.094198	0.103478	0.103843	0.853020	0.037464	0.084883
L_Sa	0.000260	-0.005689	-0.000948	-0.003161	0.819240	0.002508
L_Si	0.000187	-0.001310	-0.000158	-0.000491	0.003133	0.790251

(a) Non Informative Prior - Beta Lagged Prices

	N	NC	SC	S	Sa	Si
L_N	0.786457	0.014255	0.051226	0.055937	0.047855	0.043265
L_NC	-0.006254	0.839351	-0.023313	-0.022764	-0.052787	-0.045050
L_SC	-0.095904	-0.169700	0.646626	-0.093398	-0.078665	-0.063672
L_S	0.094209	0.103295	0.103406	0.852932	0.037298	0.084891
L_Sa	0.000292	-0.005449	-0.000832	-0.003035	0.819363	0.002737
L_Si	0.000014	-0.001493	-0.000321	-0.000650	0.002961	0.790094

(b) Normal Conjugate Prior - Posterior mean - Lagged Prices

Show coefficients with a credibility interval which does not contain zero

Show coefficients with a credibility interval which contain zero

Figure 9: Comparison between the Posterior Mean of the *Non Informative P.* and *Natural Conjugate P.*

Running the regression with the parameter distribution for β and Σ as laid down in section 4.2 (*Normal Conjugate Prior*), we obtained a slight change of the coefficients scenario does not seem to result more parsimonious than the previous version.

For what concerns the lagged prices, Sardegna LP 's betas is the only one that does not affect the others because Sicilia LP beta[23] becomes significantly negative in relation to the north centre prices (even if the value of the coefficient is much lower than those of the other peninsular regions). The remaining betas did not significantly change or increase compared to the non informative ones.

[23] lagged prices

	N	NC	SC	S	Sa	Si
Bologna	0.134565	0.139770	0.139395	0.133389	0.097706	0.253810
Brescia	-0.004543	0.040651	-0.000265	0.001908	0.120719	-0.050885
Genova	0.060114	0.064739	0.054247	0.067426	-0.015220	-0.072844
Milano	0.146665	0.183148	0.174498	0.173584	0.054492	0.201987
Rimini	0.018518	0.032542	0.028128	-0.001774	0.013558	0.120029
Torino	-0.078658	-0.054833	-0.066538	-0.087323	0.076195	0.023038
Trieste	-0.061781	-0.085659	-0.079966	-0.059699	-0.112311	-0.160961
Venezia	-0.027533	-0.045730	-0.061043	-0.074802	-0.003099	-0.028408
Firenze	0.067868	0.043715	0.063746	0.055227	0.074522	0.097375
Perugia	0.015936	-0.019236	0.021555	0.013363	0.173557	0.106821
Roma	-0.135274	-0.139149	-0.157613	-0.163284	-0.086992	-0.231674
Napoli	0.183815	0.199276	0.191504	0.183704	0.153533	0.257747
Pescara	0.054306	0.056250	0.053406	0.043103	0.029899	0.257534
Bari	-0.150190	-0.181478	-0.116138	-0.119483	-0.183732	0.023499
Reg_Cal	-0.012392	-0.001399	0.030789	0.068425	0.034771	0.194378
Cagliari	0.044648	0.077544	0.024473	0.048905	-0.044691	-0.238384
Palermo	-0.201041	-0.266519	-0.227059	-0.214620	-0.150482	-0.239674
Catania	-0.073862	-0.061865	-0.068912	-0.059473	-0.131225	-0.205883

Show coefficients with a credibility interval which does not contain zero

Show coefficients with a credibility interval which contain zero

Table 6: *Normal Conjugate Prior* - Posterior Mean - Relation of Temperatures and Zones

About temperatures, the coefficient's values remain close to the ones of previous version. Perugia's beta temperature becomes non significative for all the regions included the one from where it belongs. Napoli remains the exception among the coefficients of the provinces located in the south showing significantly positive values with all the regions. In the north side the same happens in what concerns Trieste's temperatures where, but differently from before, the province coefficient loses significance with the south energetic region. Therefore, hypothetically (with the exception for the provinces listed above) I can assume that an increase of the temperature of southern regions generates diffuse decrease in electricity prices (in same cases, of whole Italy), counterbalanced (but not proportionally) by the increasing of northern region temperatures that produces a diffuse increasing of EPs.

The few variations in loads betas significance are recorded by Sardegna's demanded and offered volumes coefficients (which becomes irrelevant to Sardegna's and Sicilia's prices) and by Centre North's offered volumes beta that is not significative for South anymore. What said in the previous paragraph about the dummies also applies to the natural conjugate results.

Strict Prior

The output carry out estimating the model by imposing more strict conditions, are descriptive of a substantial changing of betas significance.

This changing does not concern the significance of the lagged zonal betas because they continue to capture the price variations like they did in the non informative ones.

Their value is slightly increased for the coefficients significantly positive and decreased for negative.

	N	**NC**	**SC**	**S**	**Sa**	**Si**
L_N	0.786656	0.014667	0.051262	0.055907	0.047668	0.044037
L_NC	-0.006321	0.839506	-0.023132	-0.022474	-0.052735	-0.044613
L_SC	-0.095805	-0.169758	0.646372	-0.093279	-0.078755	-0.063825
L_S	0.094198	0.103478	0.103843	0.853020	0.037464	0.084883
L_Sa	0.000260	-0.005689	-0.000948	-0.003161	0.819240	0.002508
L_Si	0.000187	-0.001310	-0.000158	-0.000491	0.003133	0.790251

(a) *Non Informative Prior* - Posterior Mean - Lagged Prices

	N	**NC**	**SC**	**S**	**Sa**	**Si**
L_N	0.787091	0.014891	0.051753	0.056454	0.046689	0.041666
L_NC	-0.004274	0.842377	-0.020575	-0.020446	-0.052085	-0.043618
L_SC	-0.098698	-0.173493	0.643025	-0.096623	-0.083055	-0.065238
L_S	0.095470	0.104992	0.105047	0.854384	0.039285	0.084703
L_Sa	-0.000348	-0.006438	-0.001764	-0.003858	0.822251	0.001893
L_Si	0.000631	-0.000817	0.000496	0.000085	0.003517	0.793073

(b) *Strict Prior* - Posterior Mean - Lagged Prices

Show coefficients with a credibility interval which does not contain zero

Show coefficients with a credibility interval which contain zero

Figure 10: Comparison between the posterior mean of the *Strict Prior* and the *Natural Conjugate Prior*

The constrains generate a completely different scenario, betas of temperatures like the one of Trieste, Venezia and Genova become not significative for the southern regions and also for the northern ones. The same situation is outlined in Catania temperatures coefficients for all the areas.

	N	NC	SC	S	Sa	Si
Bologna	0.046747	0.027287	0.025919	0.030337	-0.008276	0.064904
Brescia	-0.031557	0.004582	-0.023948	-0.013957	0.023522	-0.021867
Genova	0.020415	0.022175	0.010779	0.025077	-0.004911	-0.025361
Milano	0.046509	0.056598	0.041605	0.049971	-0.008787	0.032087
Rimini	.002933	0.009259	0.009571	-0.012451	0.000836	-0.014754
Torino	0-0.040961	-0.012074	-0.016274	-0.035848	0.036161	0.007920
Trieste	-0.013407	-0.021180	-0.018747	-0.010363	0.001795	-0.014724
Venezia	0.008456	-0.000569	-0.005422	-0.018154	0.009074	0.029723
Firenze	0.038965	0.008620	.022527	0.016007	0.017812	-0.000148
Perugia	-0.006266	-0.039687	-0.004845	-0.011907	0.051161	0.007048
Roma	-0.025879	-0.013148	-0.021127	-0.030750	0.014513	-0.036560
Napoli	0.048473	0.041391	0.033257	0.040705	0.009649	0.054836
Pescara	0.004562	0.002515	-0.002161	-0.009717	-0.008890	0.036911
Bari	-0.046625	-0.054289	-0.001790	-0.020125	-0.028128	0.030568
Reg_Cal	-0.027829	0.021550	-0.004247	0.024502	0.011588	0.037539
Cagliari	0.008497	-0.032087	-0.012449	0.016791	-0.018373	-0.051653
Palermo	-0.040807	–0.067330	-0.032687	-0.038665	0.004499	-0.030591
Catania	-0.018476	-0.001979	-0.004550	-0.000184	-0.004790	-0.022622

(grey)	Show coefficients with a credibility interval which does not contain zero
(white)	Show coefficients with a credibility interval which contain zero

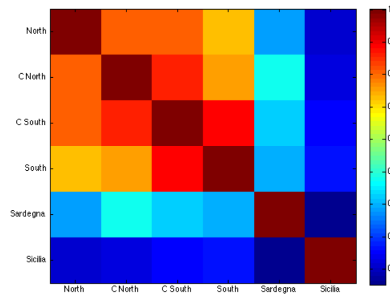
Table 8: *Strict Prior* - Posterior Mean - Relation of Temperature and Zones

Coefficients related with Sardegna become not significative with the exception of Perugia. Napoli betas continue to be significative and positive in relation to all the regions despite the restrictions imposed (Sardegna excluded). However its betas values drastically fall to $\simeq 0,04$. This decreasing in coefficient values is diffuse to all the results reported in table (8).

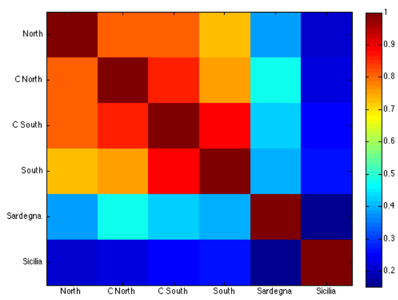
Another interesting feature of the results could be observed in the central part of the table where betas temp share the same significance and almost the same values related to both the central regions lagged prices.

The North and North Centre betas of the demanded volumes appear related just with the region poles apart. Regarding the offered volumes side the situation appears similar to the one of the first paragraph except for the coefficients of North Centre and of Sardegna that become significative for the South and for Sicilia respectively.

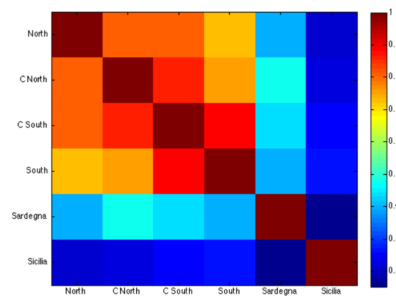
All the hourly dummies remain significative positive even with this prior setting except for the D_2, D_3, D_4, D_{22} which are non significative in some ares. What said for the daily dummies could be applied also to the weekly one because their betas remain high significantly negative.



(a) Non Informative Prior



(b) Natural Conjugate Prior



(c) Strict Prior

Figure 11: Correlation Matrix

In Figure (11) are represented the correlation matrix calculated on the sigma mean estimated by *squeezing* [24] the sigma draws carried out from the Gibbs sampler. The cold colors are index of low correlation among electricity zones. It's the case of Sicilia and Sardegna whose *rho*[25] with the other regions is close to zero and it is showed by the blue gradation.

The relation among region is well explained in all the given figures and the repetitive pattern seems to describe with great accuracy the Italian grid system where Sicilia and Sardegna are linked with the mainland just by a submarine cable when the other regions share a great number of offer point.

Except for the diagonal, the most correlated zones are the central ones with a $0.85 \leq \rho \leq 0.89$ followed by the correlation of North Centre and South Centre with the North and South with the North itself.

[24]term related with Matlab code used to obtain the Sigma posterior mean
[25]correlation coefficient

6 Conclusions and Possible Developments

In this thesis, a Bayesian model has been employed to determinate the interdependency among Italian electricity zones.

The autoregressive part, described by the lagged prices, provides a first hint on the situation of the Italian electricity market. North zone results the only zone that is not affected by the lagged prices of the other regions considering that the effects of South and South Centre lagged prices are counterbalanced. Sardegna and Sicilia coefficients do not result significative in relation with the other zones of the mainland.

As regard to the demand volumes an increase in Sicilian one generates a diffuse decrease of the electricity prices, whereas we register a countertrend for all the other regions, except for the Northern ones whose demand does not affect the price of other zones. On the offered volumes side Sicilia and South ones are not relevant on the other regions prices, the same happens for the offer of the North Centre in relation to the price of the Southern areas.

From the hourly dummies appear that the electricity prices tend to increase from early morning to afternoon then they decrease till midnight describing and capturing variations in the daily use of electricity. Furthermore weekly dummies describe the cycles in spot prices due to the variations in business or residential use that generates higher prices during the working days than in the weekends.

We estimate the Bayesian model under three different prior assumptions. With reference to the zonal interdependency, it is rather well explained by the posterior mean and variance of all the prior, even if the data are affected by high volatility.

However it is indisputable the presence of particular behavior in some variables (it is the case of Napoli, Milano and Palermo among the weather coefficients). With regards to the Strict Prior, it generates a drastic change in the significance of many coefficients but, it does not change the weather scenario completely because a common feature to all the prior used is defined by the presence of positive coefficients for the Northern provinces (above Rome) and of negative coefficients for the Southern ones (below Rome).

This model is suitable to be extended by adding a financial variable or by including a *contiguity matrix* of zonal proximities. A forecast analysis could be used to test the predictive power of each prior used.

7 Appendix A

Likelihood Function

A likelihood function $L(a)$ is the probability or probability density for the occurrence of a sample configuration$x1, ..., x_n$ given that the probability density $p(x_1; a)$ with parameter a is known,

$$L(a) = p(x_1|a) \dots p(x_n|a)$$

(Harris and Stocker 1998, p. 824).

Multivariate Normal Distribution

$$x \sim N_n(\mu, \Sigma)$$

$$p(x) = (2\pi)^{-n/2} |\Sigma|^{-1/2} exp\left[-\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu)\right]$$

Invers Wishart Distribution

The Wishart distribution is the multivariate generalization of the *gamma distribution*.

If $W \sim W(Q, q)$, where W is of dimensions$(k \times k)$, then its density is proportional to:

$$|W|^{(q-k-1)/2} \times exp(-\frac{1}{2}tr(Q^{-1}W))$$

On the other hand, if $W^{-1} \sim W(Q, q)$, then W has the inverse-Wishart distribution. The inverse-Wishart is the conjugate prior distribution for the multivariate normal covariance matrix.

$iW(W; \nu, S{-}1)$, where ν represents the degrees of freedom and S is a $(k \times k)$ symmetric, positive definite scale matrix, is given by:

$$A_n \sim W_v(S^{-1})$$

$$p(A) = \left(2^{vk/2}\pi^{k(k-1)/4}\prod_{1=1}^{k}\Gamma\left(\frac{v+1-i}{2}\right)\right)^{-1} \times |S|^{v/2} |A|^{-(v+k+1)/2} exp\left(-\frac{1}{2}tr\left(SA^{-1}\right)\right)$$

8 References

[1] Bańbura M., Giannone D. and Reichlin L. (2008), *Large Bayesian VARs,* Working paper series N° 966, European Central Bank

[2] Canova Fabio and Ciccarelli Matteo (1999), *Forecasting and turning point prediction in Bayesian Panel VAR Model*, Universitat d'Alicant

[3] Canova Fabio and Ciccarelli Matteo (2006), *Estimating multi-country VAR models*, Working paper series N° 603, European Central Bank

[4] Ciccarelli Matteo and Rebucci Alessandro (2003), *Bayesian VARs: A survey of the recent literature with an application to the European Monetary System,* Research Department, Inernationl Monetary Fund (IMF) Working Paper

[5] Granger C. W. J. (1969), *Investigating causal relations by econometric models and cross-spectral methods*, Econometrica, vol. 37, no. 3 pp. 424–438

[6] Koop Gary (2003), *Bayesian Econometrics*, published by Jonh Wiley and Sons Ltd

[7] Koop Gary and Korobilis Dimitris (2009), *Bayesian Multivariate Time Series Methods for Empirical Macroeconomics*, University of Strathclyde

[8] Gestore Mercati Elettrici s.p.a., *Vademecum of the Italian Power Exchange*

[9] Harris, J. W. and Stocker, H. (2006), *Handbook of Mathematics and Computational Science*, p. 824

[10] Griffiths William E. (2001), *Bayesian Inference in the Seemingly Unrelated Regressions Model*, Economics Department University of Melbourne

[11] Le Sage James P. (1998), *Spatial Econometrics,* Department of Economics University of Toledo

[12] Popova Julia (2004), *Spatial pattern in modelling electricity prices: evidence from the PJM market*, West Virginia University

[13] Lancaster Tony (2004), *An Introduction to Modern Bayesian Econometrics*, West Virginia University, Blackwells

[14] Serati Massimiliano, Manera Matteo and Plotegher Michele (2007), *Modelling Electricity Prices: from a state of the art to a draft of a new proposal,* Liuc Papers N° 210, Serie Economia ed Impresa

[15] Xia Youlong, Fabian Peter, Andreas Stohl and Winterhalter Martin (1999), *Forest climatology: estimation of missing values for Bavaria, Germany*, Ludvig-Maximilians Universitat Munchen

Data retrieving

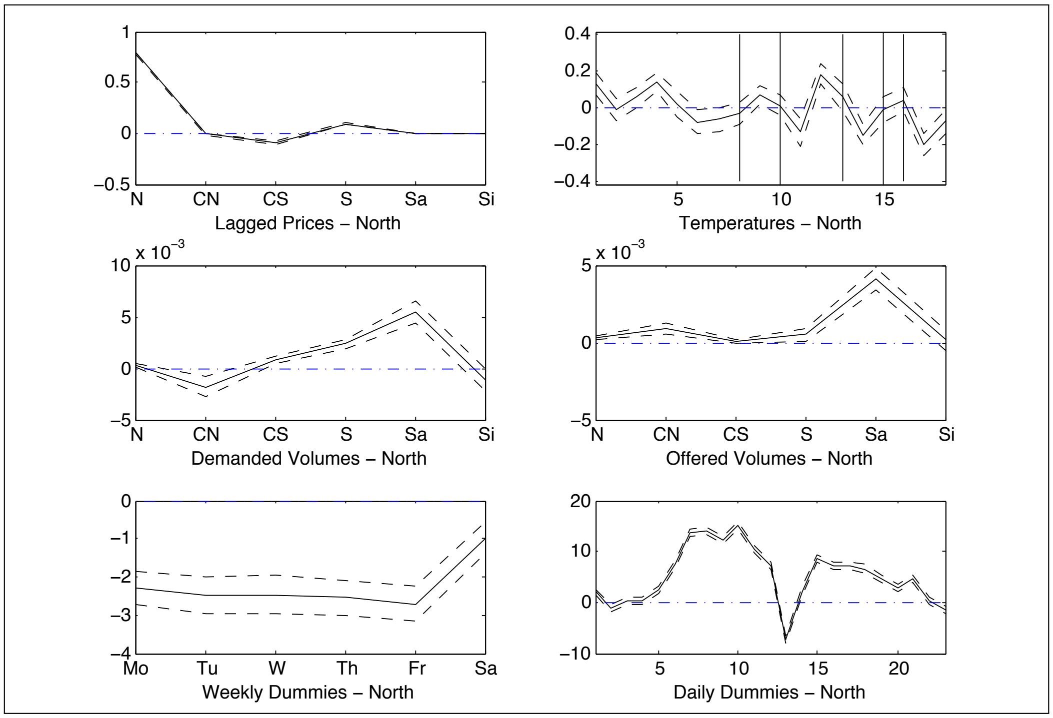
[1] *www.mercatoelettrico.org* - Electricity Spot Prices (Hourly - 2008:2011), Volumes (Hourly - 2008:2011)

[2] *www.wunderground.com* - Temperatures of 18 Italian Provinces (Hourly - 2008:2011)

Data analysis was carried out with MATLAB

9 Appendix B (*Graphs and Tables*)

Non Informative Prior



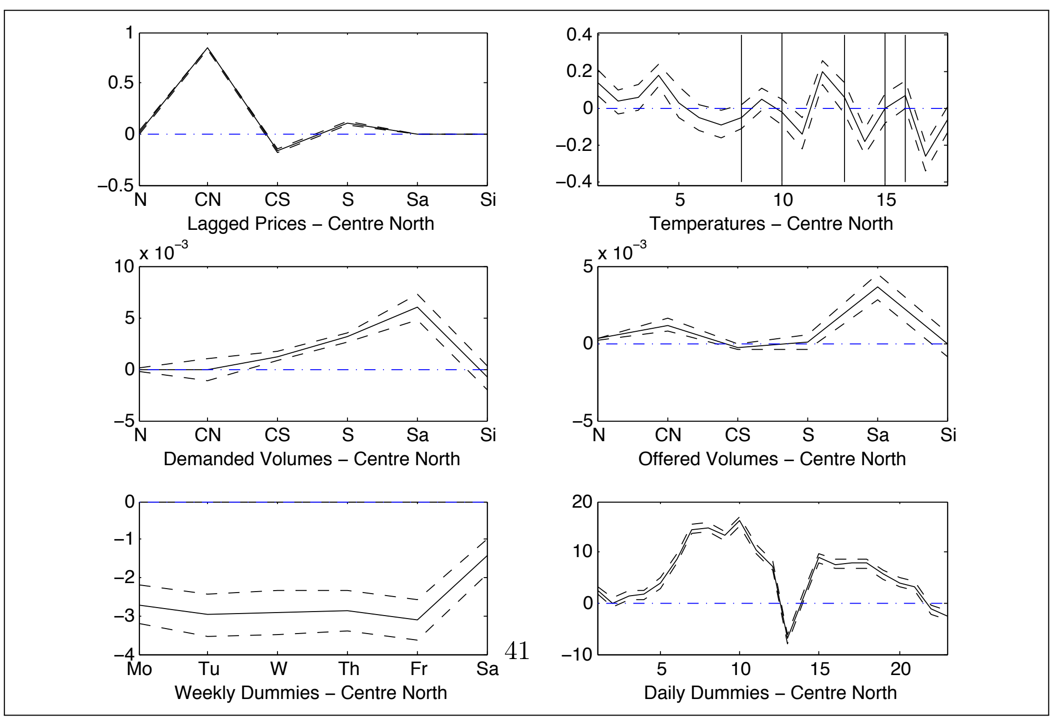


Figure 12: Non Informative Prior - *North* & *Centre North*

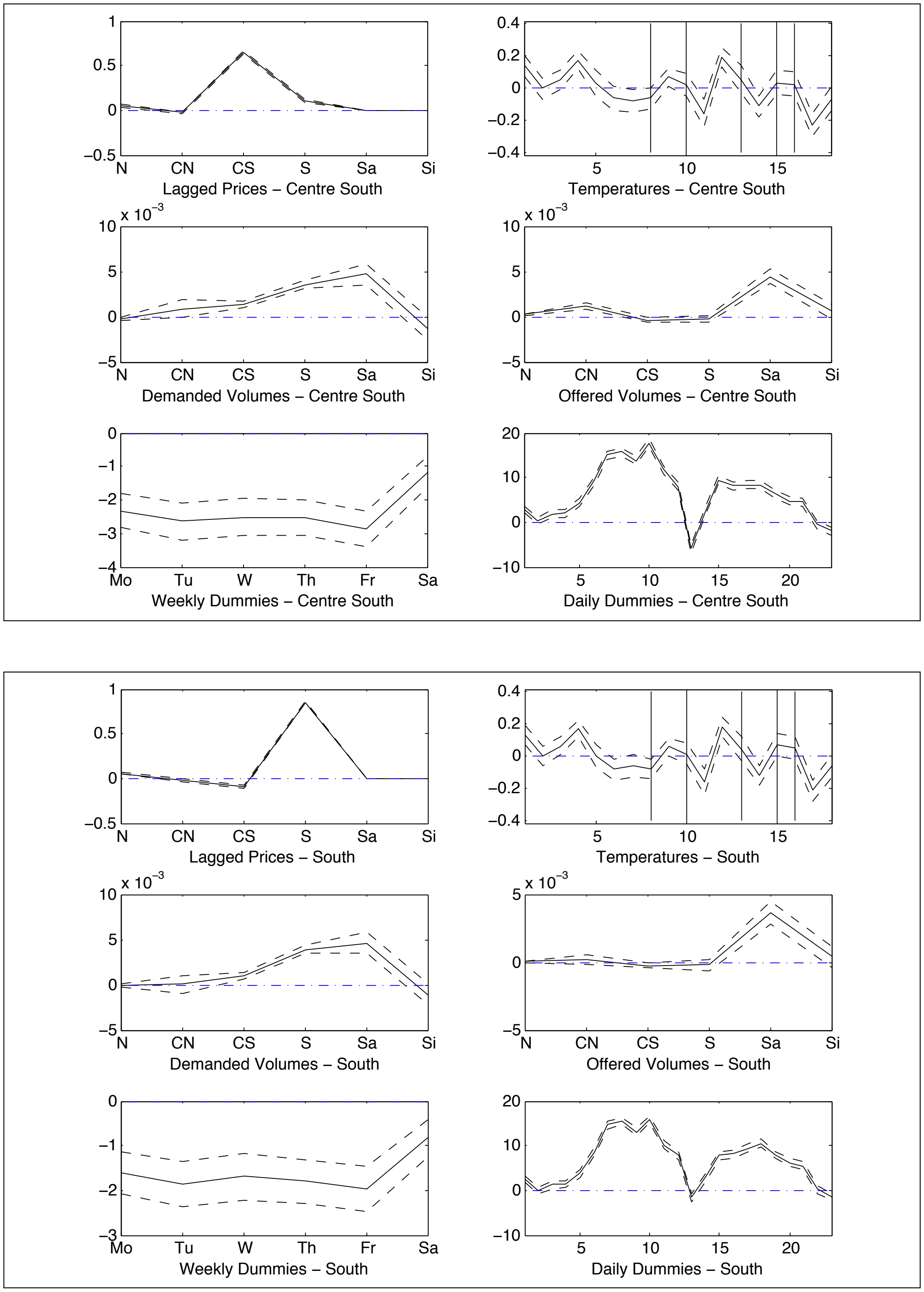


Figure 13: Non Informative Prior - *Centre South* & *South*

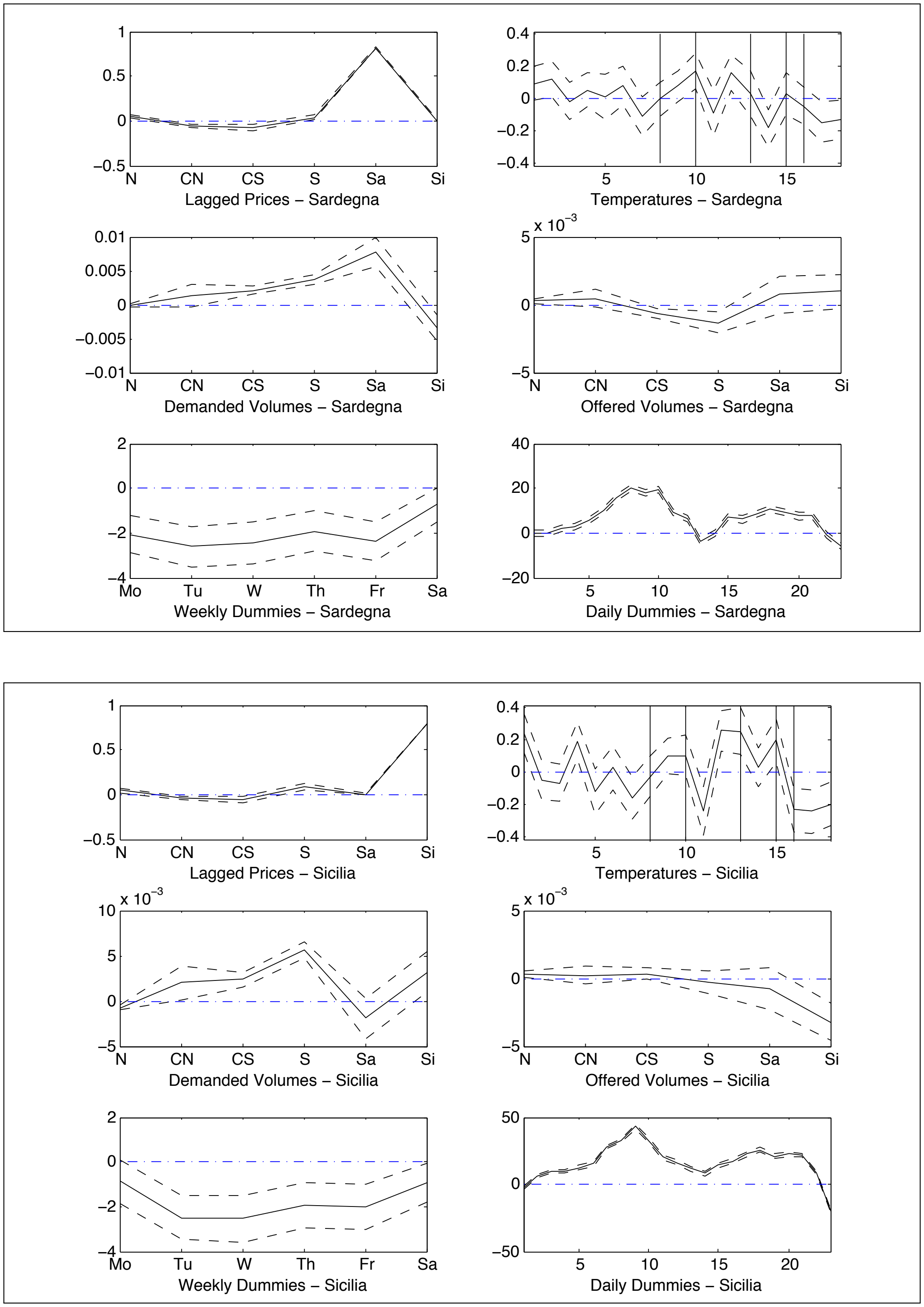
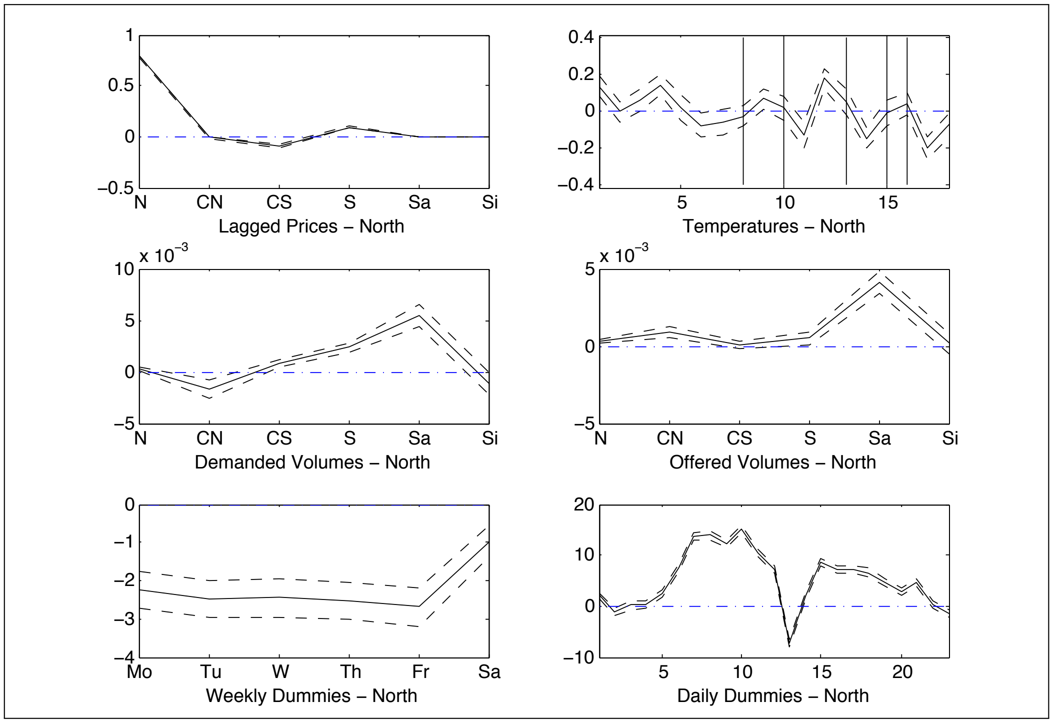


Figure 14: Non Informative Prior - *Sardegna* & *Sicilia*

Normal Conjugate Prior



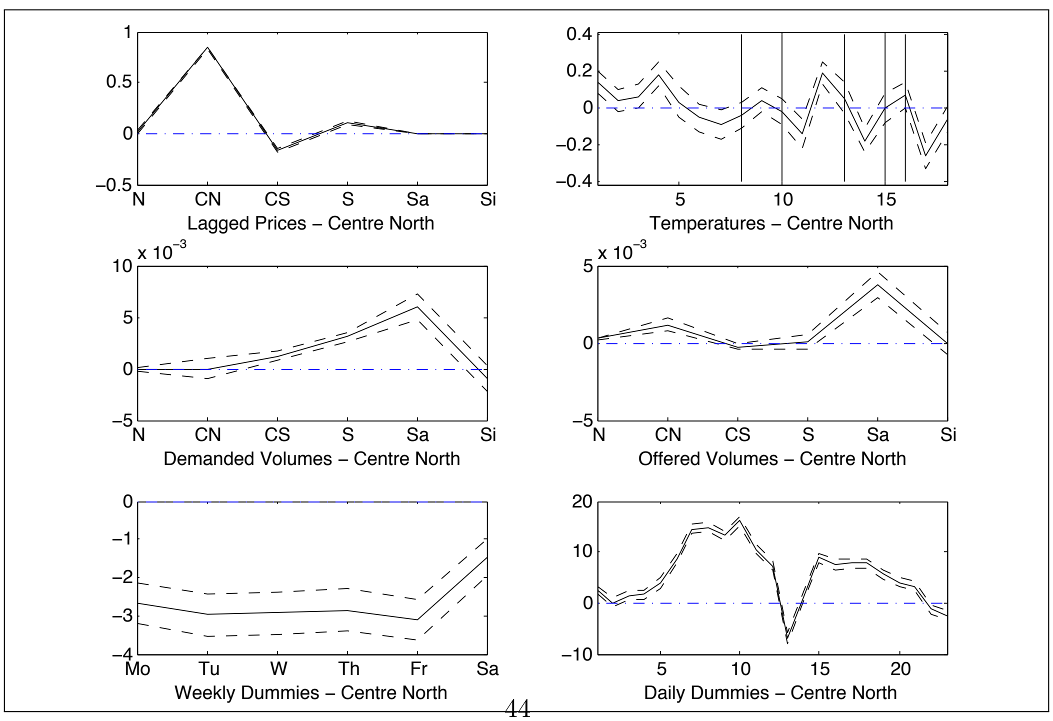


Figure 15: Normal Conjugate Prior - *North* & *Centre North*

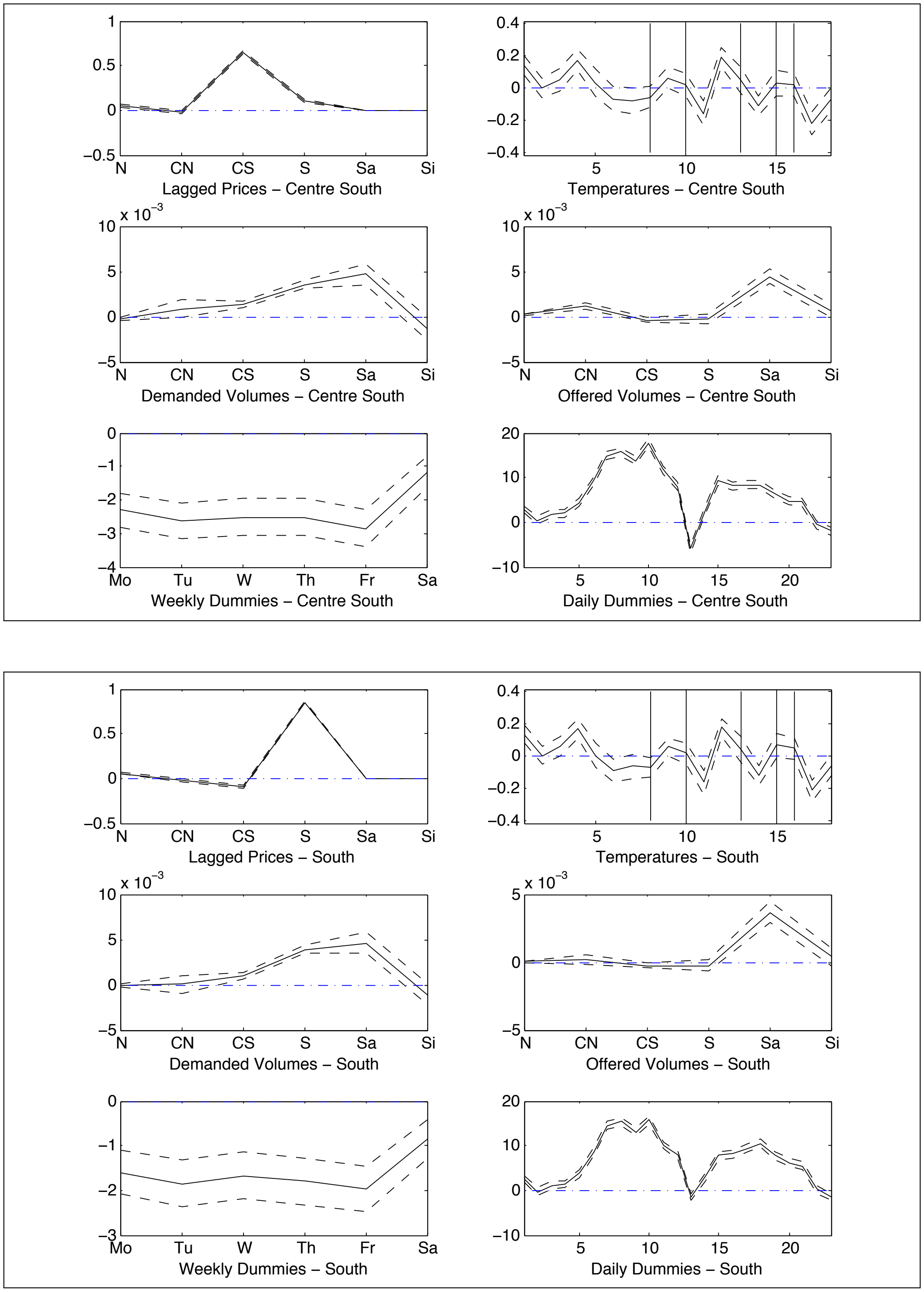


Figure 16: Normal Conjugate Prior - *Centre South* & *South*

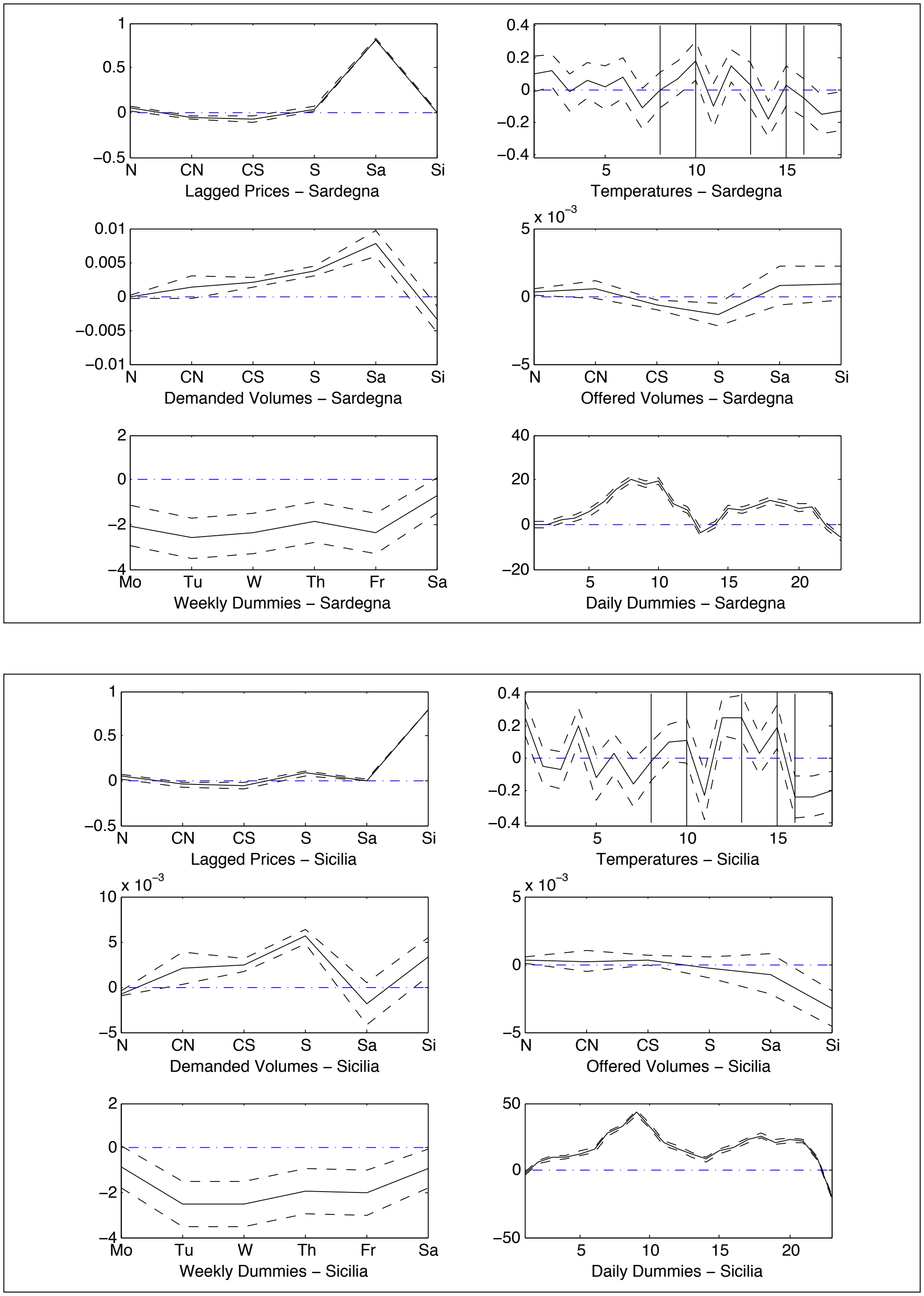


Figure 17: Normal Conjugate Prior - *Sardegna* & *Sicilia*

Strict Prior

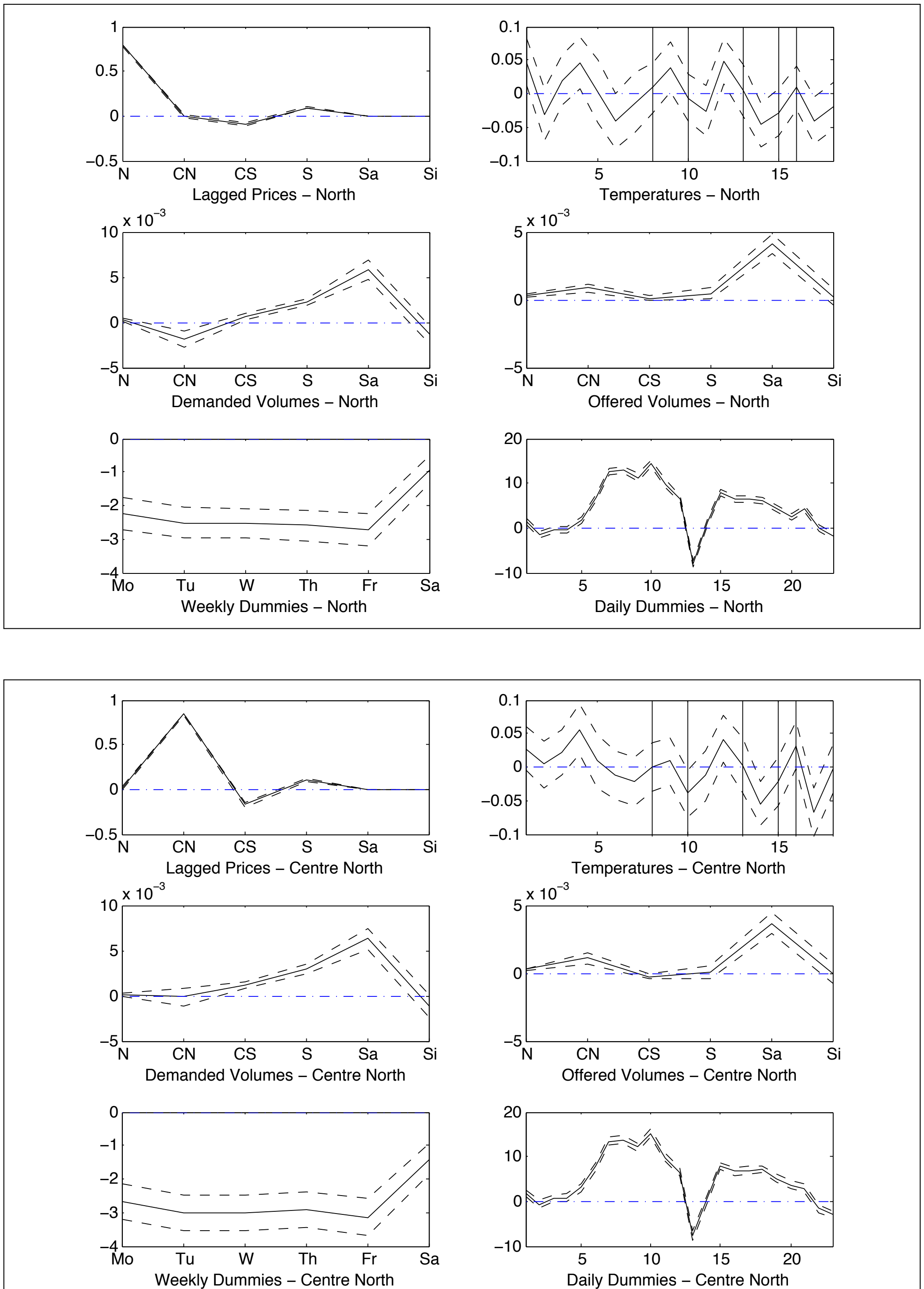


Figure 18: Strict Prior - *North* & *Centre North*

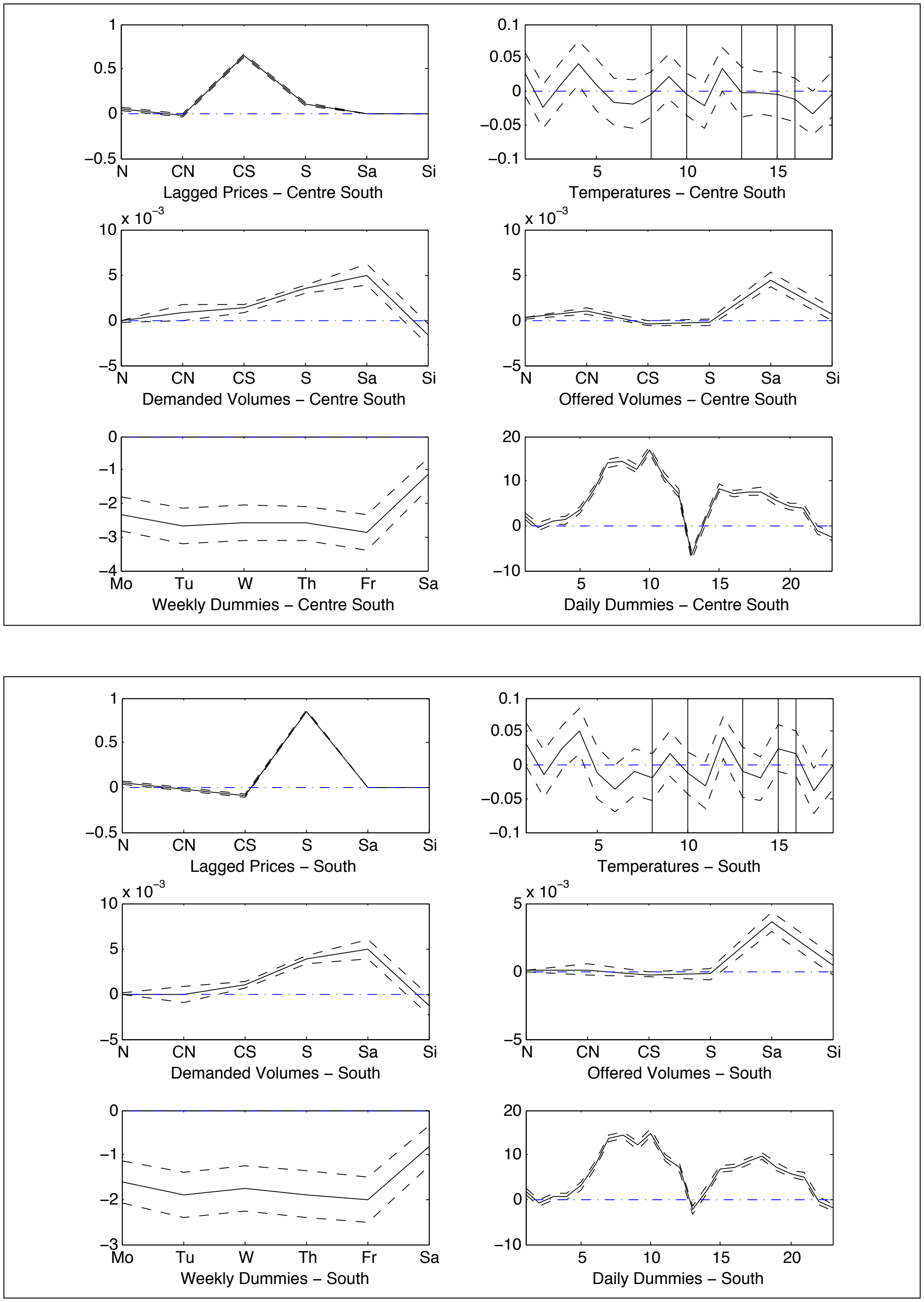


Figure 19: Strict Prior - *Centre South* & *South*

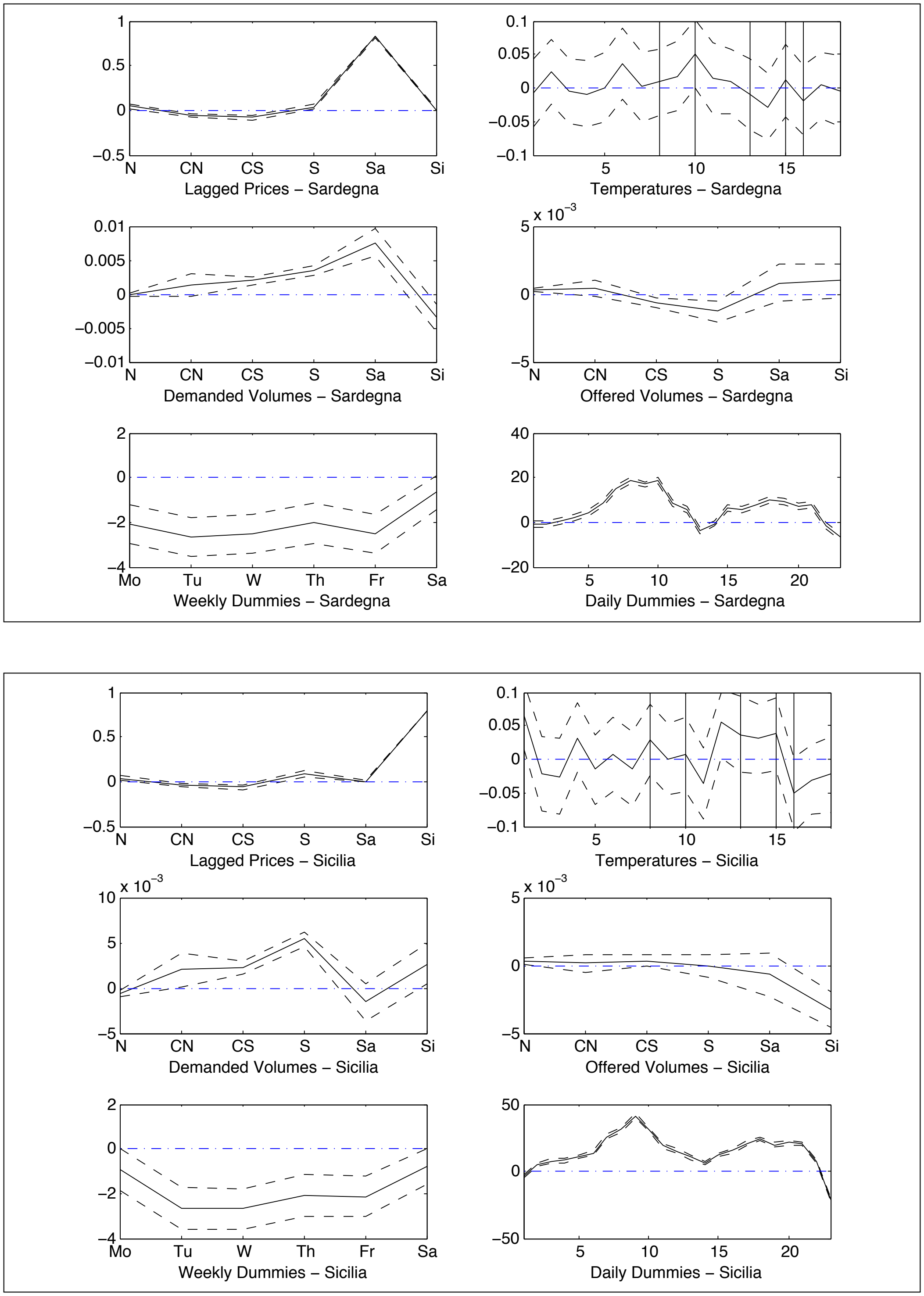


Figure 20: Strict Prior - *Sardegna* & *Sicilia*

Table 9: **Non Informative Prior - Posterior Mean**

	North	N_Centre	S_Centre	South	Sardegna	Sicilia
Constant	-15.534437	-20.490459	-20.704559	-16.101427	-20.459472	-23.759584
Lag_North	0.786656	0.014667	0.051262	0.055907	0.047668	0.044037
Lag_N_Centre	-0.006321	0.839506	-0.023132	-0.022474	-0.052735	-0.044613
Lag_S_Centre	-0.095805	-0.169758	0.646372	-0.093279	-0.078755	-0.063825
Lag_South	0.094198	0.103478	0.103843	0.853020	0.037464	0.084883
Lag_Sardegna	0.000260	-0.005689	-0.000948	-0.003161	0.819240	0.002508
Lag_Sicilia	0.000187	-0.001310	-0.000158	-0.000491	0.003133	0.790251
T_Bologna	0.135426	0.140840	0.140431	0.136542	0.102341	0.251934
T_Brescia	-0.004505	0.040576	-0.000696	0.002509	0.122437	-0.048769
T_Genova	0.057909	0.062450	0.050846	0.064274	-0.013470	-0.075861
T_Milano	0.146650	0.184483	0.174758	0.172834	0.058240	0.199955
T_Rimini	0.020528	0.034278	0.031859	0.003263	0.016034	-0.120713
T_Torino	-0.078177	-0.053990	-0.066346	-0.087594	0.076559	0.025430
T_Trieste	-0.061890	-0.087163	-0.081372	-0.062508	-0.115006	-0.159496
T_Venezia	-0.025992	-0.043725	-0.058470	-0.072602	-0.000522	-0.023310
T_Firenze	0.068910	0.044952	0.063828	0.056073	0.072906	0.098447
T_Perugia	0.017751	-0.016958	0.023071	0.015189	0.183209	0.107897
T_Roma	-0.135716	-0.140327	-0.158270	-0.163440	-0.096174	-0.233462
T_Napoli	0.180555	0.195615	0.188458	0.181355	0.153701	0.257061
T_Pescara	0.053319	0.054762	0.052254	0.041134	0.027940	0.254655
T_Bari	-0.147389	-0.178772	-0.113518	-0.118499	-0.181744	0.025355
T_Reg_Cal	-0.012079	0.000196	0.032317	0.068868	0.027916	0.195999
T_Cagliari	0.041097	0.072879	0.021279	0.046643	-0.049963	-0.242790
T_Palermo	-0.200879	-0.265178	-0.226696	-0.216144	-0.150067	-0.239205
T_Catania	-0.074174	-0.062755	-0.069663	-0.059405	-0.133372	-0.206837
D_Vol_N	0.000254	-0.000012	-0.000224	-0.000061	-0.000187	-0.000746
D_Vol_CN	-0.001772	-0.000042	0.000853	0.000003	0.001260	0.001985
D_Vol_CS	0.000770	0.001225	0.001318	0.001008	0.002066	0.002340
D_Vol_S	0.002349	0.003076	0.003557	0.003875	0.003663	0.005599
D_Vol_Sa	0.005504	0.006002	0.004663	0.004623	0.007772	-0.001890
D_Vol_Si	-0.001202	-0.000933	-0.001360	-0.001111	-0.003443	0.003238
O_Vol_N	0.000312	0.000243	0.000222	0.000011	0.000291	0.000302

O_Vol_CN	0.000913	0.001190	0.001116	0.000178	0.000481	0.000233
O_Vol_CS	0.000056	-0.000249	-0.000373	-0.000268	-0.000667	0.000326
O_Vol_S	0.000491	0.000083	-0.000269	-0.000244	-0.001373	-0.000257
O_Vol_Sa	0.004139	0.003714	0.004464	0.003637	0.000793	-0.000727
O_Vol_Si	0.000134	-0.000069	0.000558	0.000370	0.000934	-0.003294
D_Week_Mo	-2.244664	-2.687596	-2.310346	-1.602761	-2.054250	-0.853054
D_Week_Tu	-2.482299	-2.979909	-2.626312	-1.845970	-2.615231	-2.493627
D_Week_W	-2.455901	-2.928412	-2.506655	-1.672568	-2.399328	-2.534267
D_Week_Th	-2.529730	-2.854631	-2.509506	-1.793956	-1.902139	-1.934018
D_Week_Fr	-2.698784	-3.112289	-2.847198	-1.978812	-2.388195	-2.011856
D_Week_Sa	-0.968287	-1.458063	-1.171702	-0.843676	-0.719891	-0.915068
D_Day_1	1.933812	2.460585	2.799711	2.462389	0.013809	-2.295756
D_Day_2	-1.080583	0.044862	0.386342	-0.245660	-0.065779	6.141904
D_Day_3	0.156747	1.467494	1.832563	1.195913	1.767227	9.254907
D_Day_4	0.188216	1.630573	2.028657	1.407384	2.804025	9.623996
D_Day_5	2.511903	3.890431	4.372139	3.821916	5.327814	12.348763
D_Day_6	7.437846	8.714579	9.190162	8.692449	9.672382	15.190618
D_Day_7	13.551175	14.419992	14.928437	14.561001	15.954190	27.509287
D_Day_8	13.855662	14.884947	15.700916	15.404333	19.998193	32.694587
D_Day_9	12.205615	13.221643	13.775437	13.084034	18.069257	43.118612
D_Day_10	15.058232	16.091803	17.790704	15.773926	19.219506	33.021432
D_Day_11	10.506884	10.531224	11.692766	10.099760	9.409097	20.119237
D_Day_12	7.082978	7.226842	8.012182	7.810835	6.281761	15.980449
D_Day_13	-7.367935	-6.956448	-5.985169	-1.578129	-3.442356	11.881142
D_Day_14	2.220516	1.428580	2.512669	3.050950	0.110298	7.978727
D_Day_15	8.554934	8.842519	9.347627	7.767700	6.887107	13.995911
D_Day_16	7.191238	7.536482	8.120302	8.084814	6.072938	17.107167
D_Day_17	7.232380	7.755304	8.424257	9.479483	8.524366	22.719714
D_Day_18	6.592019	7.835764	8.421905	10.492702	10.727574	25.745459
D_Day_19	4.715694	5.646598	6.275060	7.833852	9.477111	20.836913
D_Day_20	2.738208	4.041999	4.798415	6.219766	7.463238	22.588283
D_Day_21	4.551994	3.308378	4.466414	5.528022	7.895389	21.581858
D_Day_22	0.298546	-1.241889	-0.533949	0.130414	-0.838247	8.203239
D_Day_23	-1.397072	-2.441005	-1.973953	-1.479542	-6.150044	-20.399352

Table: **Normal Conjugate Prior - Posterior Mean**

	North	N_Centre	S_Centre	South	Sardegna	Sicilia
Constant	-15.534660	-20.479262	-20.719949	-16.176847	-20.580222	-23.728536
Lag_North	0.786457	0.014255	0.051226	0.055937	0.047855	0.043265
Lag_N_Centre	-0.006254	0.839351	-0.023313	-0.022764	-0.052787	-0.045050
Lag_S_Centre	-0.095904	-0.169700	0.646626	-0.093398	-0.078665	-0.063672
Lag_South	0.094209	0.103295	0.103406	0.852932	0.037298	0.084891
Lag_Sardegna	0.000292	-0.005449	-0.000832	-0.003035	0.819363	0.002737
Lag_Sicilia	0.000014	-0.001493	-0.000321	-0.000650	0.002961	0.790094
T_Bologna	0.134565	0.139770	0.139395	0.133389	0.097706	0.253810
T_Brescia	-0.004543	0.040651	-0.000265	0.001908	0.120719	-0.050885
T_Genova	0.060114	0.064739	0.054247	0.067426	-0.015220	-0.072844
T_Milano	0.146665	0.183148	0.174498	0.173584	0.054492	0.201987
T_Rimini	0.018518	0.032542	0.028128	-0.001774	0.013558	0.120029
T_Torino	-0.078658	-0.054833	-0.066538	-0.087323	0.076195	0.023038
T_Trieste	-0.061781	-0.085659	-0.079966	-0.059699	-0.112311	-0.160961
T_Venezia	-0.027533	-0.045730	-0.061043	-0.074802	-0.003099	-0.028408
T_Firenze	0.067868	0.043715	0.063746	0.055227	0.074522	0.097375
T_Perugia	0.015936	-0.019236	0.021555	0.013363	0.173557	0.106821
T_Roma	-0.135274	-0.139149	-0.157613	-0.163284	-0.086992	-0.231674
T_Napoli	0.183815	0.199276	0.191504	0.183704	0.153533	0.257747
T_Pescara	0.054306	0.056250	0.053406	0.043103	0.029899	0.257534
T_Bari	-0.150190	-0.181478	-0.116138	-0.119483	-0.183732	0.023499
T_Reg_Cal	-0.012392	-0.001399	0.030789	0.068425	0.034771	0.194378
T_Cagliari	0.044648	0.077544	0.024473	0.048905	-0.044691	-0.238384
T_Palermo	-0.201041	-0.266519	-0.227059	-0.214620	-0.150482	-0.239674
T_Catania	-0.073862	-0.061865	-0.068912	-0.059473	-0.131225	-0.205883
D_Vol_N	0.000250	-0.000015	-0.000230	-0.000067	-0.000181	-0.000744
D_Vol_CN	-0.001769	-0.000021	0.000885	0.000020	0.001246	0.002031
D_Vol_CS	0.000774	0.001225	0.001326	0.001015	0.002094	0.002320
D_Vol_S	0.002361	0.003079	0.003565	0.003880	0.003685	0.005589
D_Vol_Sa	0.005518	0.005995	0.004635	0.004610	0.007787	-0.001921
D_Vol_Si	-0.001209	-0.000923	-0.001363	-0.001103	-0.003491	0.003225

O_Vol_N	0.000314	0.000243	0.000222	0.000013	0.000280	0.000299
O_Vol_CN	0.000920	0.001204	0.001131	0.000200	0.000493	0.000247
O_Vol_CS	0.000057	-0.000252	-0.000374	-0.000266	-0.000676	0.000339
O_Vol_S	0.000492	0.000084	-0.000251	0.000228	-0.001325	-0.000248
O_Vol_Sa	0.004125	0.003719	0.004452	0.003649	0.000834	-0.000731
O_Vol_Si	0.000132	-0.000073	0.000564	0.000365	0.000911	-0.003295
D_Week_Mo	-2.258616	-2.691871	-2.318599	-1.603046	-2.059764	-0.885096
D_Week_Tu	-2.482908	-2.965409	-2.621487	-1.847193	-2.576901	-2.508925
D_Week_W	-2.474819	-2.936951	-2.520856	-1.682892	-2.414996	-2.545114
D_Week_Th	-2.538439	-2.867021	-2.520301	-1.799484	-1.914207	-1.958091
D_Week_Fr	-2.715887	-3.127159	-2.863973	-1.988403	-2.391130	-2.044268
D_Week_Sa	-0.969387	-1.435537	-1.164566	-0.829390	-0.711486	-0.903342
D_Day_1	1.918264	2.445353	2.813748	2.487766	0.004193	-2.334395
D_Day_2	-1.092053	0.018843	0.372856	-0.243795	-0.082854	6.153013
D_Day_3	0.128633	1.429588	1.814178	1.189567	1.749160	9.244014
D_Day_4	0.161728	1.580808	2.028431	1.419692	2.761648	9.583472
D_Day_5	2.498112	3.878251	4.385705	3.846951	5.313023	12.280476
D_Day_6	7.377806	8.623043	9.136338	8.662924	9.655979	15.157958
D_Day_7	13.535539	14.381300	14.918522	14.562960	15.933006	27.486141
D_Day_8	13.854648	14.856006	15.698702	15.412631	19.953152	32.654310
D_Day_9	12.187422	13.212522	13.773592	13.091025	18.100493	43.132801
D_Day_10	15.040893	16.064470	17.778951	15.777153	19.192242	33.031021
D_Day_11	10.499919	10.519834	11.679442	10.116301	9.421152	20.159494
D_Day_12	7.067872	7.207120	7.989724	7.814595	6.294299	16.033148
D_Day_13	-7.391347	-6.987010	-6.016756	-1.597117	-3.509639	11.892399
D_Day_14	2.203002	1.402397	2.501232	3.058774	0.103378	7.958071
D_Day_15	8.537128	8.828943	9.327512	7.769035	6.915709	13.945749
D_Day_16	7.185918	7.511468	8.084138	8.063579	6.076986	17.082312
D_Day_17	7.230310	7.741721	8.422862	9.485690	8.581573	22.680374
D_Day_18	6.602372	7.832694	8.416359	10.499861	10.750397	25.751564
D_Day_19	4.702655	5.604620	6.243604	7.826095	9.421867	20.812843
D_Day_20	2.697133	3.982480	4.747488	6.178695	7.447241	22.578819
D_Day_21	4.509468	3.247402	4.435240	5.516889	7.876121	21.596443
D_Day_22	0.265532	-1.292579	-0.570863	0.104439	-0.874989	8.219547
D_Day_23	-1.413276	-2.473641	-1.988256	-1.472208	-6.112777	-20.429050

Table: **Strict Prior - Posterior Mean**

	North	N_Centre	S_Centre	South	Sardegna	Sicilia
Constant	-14.696249	-19.493391	-19.753623	-15.322370	-19.262415	-22.570768
Lag_North	0.787091	0.014891	0.051753	0.056454	0.046689	0.041666
Lag_N_Centre	-0.004274	0.842377	-0.020575	-0.020446	-0.052085	-0.043618
Lag_S_Centre	-0.098698	-0.173493	0.643025	-0.096623	-0.083055	-0.065238
Lag_South	0.095470	0.104992	0.105047	0.854384	0.039285	0.084703
Lag_Sardegna	-0.000348	-0.006438	-0.001764	-0.003858	0.822251	0.001893
Lag_Sicilia	0.000631	-0.000817	0.000496	0.000085	0.003517	0.793073
T_Bologna	0.046747	0.027287	0.025919	0.030337	-0.008276	0.064904
T_Brescia	-0.031557	0.004582	-0.023948	-0.013957	0.023522	-0.021867
T_Genova	0.020415	0.022175	0.010779	0.025077	-0.004911	-0.025361
T_Milano	0.046509	0.056598	0.041605	0.049971	-0.008787	0.032087
T_Rimini	.002933	0.009259	0.009571	-0.012451	0.000836	-0.014754
T_Torino	0-0.040961	-0.012074	-0.016274	-0.035848	0.036161	0.007920
T_Trieste	-0.013407	-0.021180	-0.018747	-0.010363	0.001795	-0.014724
T_Venezia	0.008456	-0.000569	-0.005422	-0.018154	0.009074	0.029723
T_Firenze	0.038965	0.008620	.022527	0.016007	0.017812	-0.000148
T_Perugia	-0.006266	-0.039687	-0.004845	-0.011907	0.051161	0.007048
T_Roma	-0.025879	-0.013148	-0.021127	-0.030750	0.014513	-0.036560
T_Napoli	0.048473	0.041391	0.033257	0.040705	0.009649	0.054836
T_Pescara	0.004562	0.002515	-0.002161	-0.009717	-0.008890	0.036911
T_Bari	-0.046625	-0.054289	-0.001790	-0.020125	-0.028128	0.030568
T_Reg_Cal	-0.027829	0.021550	-0.004247	0.024502	0.011588	0.037539
T_Cagliari	0.008497	-0.032087	-0.012449	0.016791	-0.018373	-0.051653
T_Palermo	-0.040807	–0.067330	-0.032687	-0.038665	0.004499	-0.030591
T_Catania	-0.018476	-0.001979	-0.004550	-0.000184	-0.004790	-0.022622
D_Vol_N	0.000297	0.000039	-0.000175	-0.000017	-0.000145	-0.000667
D_Vol_CN	-0.001846	-0.000119	0.000791	-0.000055	0.001299	0.002023
D_Vol_CS	0.000706	0.001143	0.001252	0.000943	0.001963	0.002221
D_Vol_S	0.002253	0.002948	0.003447	0.003763	0.003503	0.005381
D_Vol_Sa	0.005729	0.006309	0.004963	0.004895	0.007519	-0.001607
D_Vol_Si	-0.001394	-0.001153	-0.001645	-0.001334	-0.003554	0.002637

O_Vol_N	0.000312	0.000242	0.000219	0.000010	0.000291	0.000296
O_Vol_CN	0.000842	0.001111	0.001033	0.000111	0.000395	0.000121
O_Vol_CS	0.000067	-0.000241	-0.000359	-0.000251	-0.000657	0.000355
O_Vol_S	0.000471	0.000059	-0.000270	-0.000236	-0.001306	-0.000036
O_Vol_Sa	0.004091	0.003657	0.004420	0.003610	0.000820	-0.000712
O_Vol_Si	0.000179	-0.000011	0.000617	0.000415	0.000956	-0.003285
D_Week_Mo	-2.240328	-2.677161	-2.316211	-1.609845	-2.087860	-0.928902
D_Week_Tu	-2.514624	-3.008088	-2.665939	-1.895454	-2.643919	-2.656033
D_Week_W	-2.533206	-3.009151	-2.596584	-1.762737	-2.518707	-2.690138
D_Week_Th	-2.596230	-2.932967	-2.598567	-1.879536	-2.036852	-2.081277
D_Week_Fr	-2.721803	-3.134683	-2.881733	-2.019898	-2.503444	-2.123074
D_Week_Sa	-0.932559	-1.399142	-1.120653	-0.801655	-0.645380	-0.792944
D_Day_1	1.378773	1.818249	2.148080	1.881072	-0.800998	-3.629367
D_Day_2	-1.673827	-0.684958	-0.341735	-0.904808	-0.748713	4.751926
D_Day_3	-0.479056	0.719089	1.077791	0.531791	0.899724	7.771489
D_Day_4	-0.472563	0.836637	1.256260	0.734788	1.854790	8.076038
D_Day_5	1.838114	3.090113	3.563758	3.105350	4.311255	10.777978
D_Day_6	6.684148	7.809377	8.293290	7.899173	8.567051	13.587287
D_Day_7	12.738532	13.420024	13.938022	13.668530	14.772507	25.784760
D_Day_8	12.914829	13.727101	14.548055	14.352701	18.679730	30.869122
D_Day_9	11.315576	12.151252	12.695062	12.087856	16.884540	41.411279
D_Day_10	14.307811	15.198672	16.868692	14.915353	18.344225	31.560033
D_Day_11	9.868194	9.757783	10.880878	9.354678	8.732535	18.747596
D_Day_12	6.481867	6.515451	7.244824	7.108150	5.739178	14.697431
D_Day_13	-7.958903	-7.668104	-6.733383	-2.269759	-3.982998	10.466171
D_Day_14	1.519811	0.577411	1.605701	2.209857	-0.492927	6.288789
D_Day_15	7.779874	7.918077	8.374493	6.873365	6.173769	12.119118
D_Day_16	6.457561	6.660769	7.177328	7.207243	5.389765	15.281129
D_Day_17	6.521100	6.906713	7.538662	8.661468	7.879223	20.913638
D_Day_18	6.032657	7.143875	7.698838	9.819817	10.186373	24.242126
D_Day_19	4.251000	5.083883	5.690999	7.304461	9.038478	19.563543
D_Day_20	2.376257	3.593999	4.327107	5.779709	7.151609	21.477831
D_Day_21	4.217051	2.899868	4.051366	5.143362	7.510437	20.591205
D_Day_22	-0.059016	-1.676446	-0.982053	-0.292752	-1.289084	7.159917
D_Day_23	-1.823050	-2.935479	-2.482895	-1.921223	-6.666077	-21.464631

Matlab Code used for Fourier Analysis and Missing Data

```
% Temperature Missing Values _ North
y=Temperature18(:,1:end);
yy=y;
k=8;
j=1;
for i=j:k
    alpha=0.05;
    n=size(y,1);
    X1=[ones(n,1) y(:,((j:k)~=i))];
    [betahat, Ibeta, res, Ires, stat]=regress(y(:,i),X1,alpha);
    yhat=X1*betahat;
    isn=isnan(y(:,j:k));
    av=and(isn(:,i),not(any(isn(:,((j:k)~=i))')'));
    yy(av,i)=yhat(av,1);
end

%Fourier Analysis - Electricity Prices
y=Prezzi(:,:);
T=size(y,1);
I=size(y,2);
lff=zeros(T,I);
for i=1:I
    ff=abs(fft(y(:,1))).^2/(2*pi*T);
    lff(:,i)=log(ff);
end

%Plot Fourier Analysis - Italian Zones
figure
for i=1:6
subplot(3,2,i)
plot(y(:,1));
plot(2*pi*(1:T)'/T,lff(:,i));
xlim([0,2*pi]);
ylim([-10,15]);
end
```