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Ca' Foscari
Dorsoduro 3246
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Master's Degree programme – Second Cycle (*D.M. 270/2004*)

in Economia e Finanza – Economics, Econometrics and Finance

Final Thesis

Market Risk Measure and Portfolio Optimization for managing Central's Bank Portfolio

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Academic Year

2017 / 2018

DEDICATIONS

To my dear parents

MOÏNDI NDACKO Victorien and MOÏNDI YANGBANGBO Célestine

To my dear brothers

MOÏNDI Manassé Cidrolin Stevens and MOÏNDI Élysée De Gad

And to my dear sisters

MOÏNDI NDACKO Favia Othniel, MOÏNDI Shara De Chélina and MOÏNDI Gracia Amenda

ACKOWNLEDGEMENTS

Throughout the academic year, previously held in the AMSE and Ca' Foscari and the writing of this thesis, we have never ceased to receive support of all kinds. We therefore, by these few words, for what they are worth, express our gratitude.

We would like to thank our supervisor Prof. Marco CORAZZA, who, in spite of all the responsibilities that are his, agreed to support us. This document could not have been published without its practical advice, both in substance and in form. We express our gratitude. We thank also Monica BILLIO, professor at Ca 'foscari and Eric GIRARDIN, professor at AMSE for coordinating our Erasmus exchange.

A special mention for all the teaching staff of AMSE, and also Ca' Foscari for the quality of the teachings they have hitherto lavished on us and for their constant listening to the students.

We thank all of people of my family in France for their help and attention to our eyes when we arrived and during our stay.

CONTENTS

ABBREVIATIONS AND ACRONYMS

APT	Arbitrage Pricing Theory
BEAC	Bank of Central African States
BDF	Bank of France
WB	World Bank
CAPM	Capital Asset Pricing Model
CaR	Capital at Risk
CHF	Swiss Franc
CVaR	Conditional Value at Risk
DOF	Direction of Financial Operations
EUR	Euro
FCFA	Franc of Financial Cooperation in Africa
IMF	International Monetary Fund
GBP	Great Britain Pound
IDA	International Development Association
MC	Monte Carlo
MTM	Mark to Market
MV	Mean-Variance
SAA	Asset Allocation Strategies
USD	American Dollar
VaR	Value at Risk
XAF	Franc CFA

LIST OF TABLES

LIST OF FIGURES

FOREWORD

Aix-Marseille School of Economy (AMSE) is a school within the University of Aix-Marseille with students from several countries around the world. At the same time a teaching department, it is at the same time a laboratory of research which relies on two units of research, the Greqam (research group in quantitative economy of Aix-Marseille) and the Sesstim (economic sciences and of Health and Medical Information Processing, Joint Research Unit of Aix-Marseille University, National Institute of Health and Medical Research -Inserm, and Institute for Development Research - IRD). As part of the school partnerships, its students will for the most part study a semester at a foreign university. It is with this in mind that we came to study at the University of Ca' Foscari in exchange for Erasmus. AMSE's mission is to train a new generation of economists who understand the environment in which companies operate and are able to anticipate the challenges. In order to fulfill this objective, the school has four courses in Master's degree with six courses:

- Macroeconomics and development;
- Econometrics;
- Public economy;
- Finance.

The Master Finance is divided into two courses that are banking and Financial Econometrics the one to which we are enrolled and Finance and Insurance.

In order to complete our semester at Ca' Foscari University, we are required to do a master thesis.

Depending on the requirements in this area, our research has focused on a scientific approach in the face of a real concern whose issue is of real interest and requires the implementation of financial, statistical and econometric (or even economic) techniques. The theme that has been put forward for our consideration is: « ***Market Risk Measure and Portfolio Optimization for managing Central Bank's Portfolio*** ». That's what our work is all about.

ABSTRACT

This study was conducted in a context where portfolio securities were subject to financial market risks. What led the bank sold most of these securities in mid-March.

In this work, the problem we were considering was to measure the level of loss that the BEAC would suffer as a result of an exposure of its asset management portfolio to interest rate risks.

The objective of the work was to determine the position of this portfolio in relation to interest rate risks in order to ensure the choice of appropriate tools to measure these risks and to have an optimal portfolio. More specifically, it was about:

- describe the assets' portfolio of BEAC;
- valuate on a historical basis, the prices and the market value of the different lines of the assets;
- test the adequacy of this distribution to the normal law;
- find a solution of the portfolio optimization problem;
- define risk management policies against the potential risk of the portfolio.

To achieve our objectives, the methodology used was CVaR Portfolio Optimization. At the end of this work, all our objectives have been achieved. The main results of our study are therefore as follows:

- the value of the security is function of the interest rate: in case where the interest rate decrease at the time of the renewals of the falls of deadlines, the output of the wallet is negatively impact; in case of an increase of the interest rate, the value of the security is negatively impacted;
- The acquired securities on French market account for the majority of portfolio holdings with 51%;
- FRTR9 stock is the security that is most exposed to interest rate risk; for a 1% change in the rate of return;
- rates of return are not normally distributed;
- the evaluation of CVaR indicates us that the expected loss would be to 0.333 of the value of market of the portfolio.

INTRODUCTION

I. Research context and justification

One of the main purpose of financial institutions is asset management[1]. As a result, they are regularly faced with various risks. Central Banks are not exempt. Thus, Central Bank's portfolio of assets may be exposed to financial risks, including market risk, in the context of the management of its foreign exchange reserves through its Trading Room[2].

In general, financial institutions can deal with market risk, credit risk and transaction risk. Over the past ten years, in line with all developments in financial intermediation, the perception and management of risks incurred by financial institutions has changed dramatically. As a result, there has been a significant change in risk assessment and risk management methods. The measure and management of financial risk is thus one of Central Bank's major problems since it interacts on the financial market by carrying out transactions on post-balance-sheet securities, forex, etc.

In a simple way, market risk is defined as the risk of losses resulting from changes in market prices (stock prices, commodities, currencies and interest rates). Market risk is composed of several risks in its own right: liquidity risk, currency risk and interest rate risk.

Introduced by Harry Markowithz in 1952, the mean-variance approach is the most popular criteria in portfolio selection. Kwon as *Modern Portfolio Theory*, this theory has impacted on the investment practice in finance as practice tools for portfolio optimization. His theory is presented in two main concepts which are: the investor's goal is to maximize his return for any level of risk; and the risk can be reduced by creating a diversified portfolio of negatively related assets. In order words, the idea of this theory is to maximize a portfolio's expected return for given a level of volatility by altering and selecting the propositions of the various assets in the portfolio. MPT explains how to find the best possible diversification which is a widely used strategy for investors who want to minimize risk to a certain degree. Given an investor with two portfolios of equal value that offer the same expected return, this theory explains how the

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[1] Asset management referred to as financial assets also referred to as "portfolio management" is an activity that consists of managing the capital (owned or outsourced by a third party investor) in compliance with regulatory and contractual constraints, in applying the guidelines and / or investment policies defined by the holder of the assets under management, in order to obtain the best return possible according to a chosen level of risk.

[2] Room where the market operators involved in the financial markets are gathered.

investor will prefer and should select the less risk one. The process of selecting a portfolio can be divided into two stages according to Markowitz[3].

The first stage starts with observation and experience and ends with beliefs about the future performances of available securities.

The second stage starts with the relevant beliefs about future performances and ends with the choice of portfolio. The hypothesis will be explained by a maximum to guide investment. Consider the Markowitz theory, the rule that the investor does estimated expected return a desirable thing and variance of return an undesirable thing. These statements can be related with investment behavior.

In this theory Markowitz suggests that it is possible to construct an efficient frontier of optimal portfolios with the expected return for a given level of risk but it is not enough to consider the expected risk and return of one particular stock because by investing in more than one stock, an investor could obtain the benefits of diversification, particularly a reduction in the riskiness of the portfolio. The idea behind both theory and practice indicate that the variance is not a good measure[4]. We will give a large explanation in the following parts. So, some alternative risk measures have been proposed.

For several years, Value at Risk (VaR) has emerged as one of the market risk measures. It consists in estimating with a certain degree of precision (confidence level) the loss that can occur on an asset or on a portfolio of assets on a given horizon, for example 1 day, 10 days or 1 month, given an index selected confidence (95% or 99%). However, the Value at Risk estimate does not take into account the realization of extreme events worst than VaR (or similar) that could arise and result in the loss of a portfolio. Regarding this aspect, furthermore VaR could not be a coherent risk measure. Hence the need to dig deeper into this area. So, our study will explore some alternative to Value at Risk and portfolio optimization techniques.

II. Research problematic

In a world as turbulent as that of international finance, the lack of awareness of the effects of interest rate fluctuations on the management of the securities portfolio is one of the priority

[3] *Harry Markowitz, **Portfolio Selection**, The journal of Finance, Vol. No. 1. (Mar., 1952), pp. 77-91.*

[4] *Qin, Quande and Li, Li and Cheng, Shi, **A novel hybrid algorithm for mean-CVaR portfolio selection with real-world constraints,** Lecture Notes in Computer Science, 8795, 2014, pp. 319-327. ISSN 0302-9743*

of Central Bank. It's known that a good knowledge of the risks likely to arise on the financial market can bring a better readability of the financial flows and reduce certain uncertainties of potential losses that could suffer their portfolio. We can therefore ask the following question: **How to detect the market risk? How to get the optimal portfolio considering market risk exposure with the realization of extreme events?**

III. Research goal

The main objective of this thesis is to determine the position of the asset management portfolio in relation to market risks, specifically interest rate risks, in order to ensure that the appropriate tools are selected to measure these risks into account portfolio optimization. We will explore the Conditional Value at Risk proposed by Rockafellar and Uryasev, which is the conditional expectation of above the VaR. Specifically, it will be a question of:

- describe the assets' portfolio of BEAC;
- valuate on a historical basis, the prices and the market value of the different lines of the assets;
- test the adequacy of this distribution to the normal law;
- find a solution of the portfolio optimization problem;
- define risk management policies against the potential risk of the portfolio.

IV. Research hypotheses

During this study, we will focus on two assumptions:

- An appreciation of the interest rate may lead to a fall in the prices of investment securities on the market (and consequently in the market value).
- Operational management and / or active portfolio management may mitigate the level of risk exposure of the assets of this portfolio.

V. Research methodology

We will begin with a descriptive analysis of the asset management portfolio. The data used provides from the Bloomberg server of the Bank of Central African States (BEAC). In this study as we say before, we employ CVaR to measure the risk of portfolio. Given the portfolio selection model, we propose some real-world constraints. All steps of this algorithm will be implemented on Matlab and we will use also XLSTAT for the normality test and EVIEWS for some descriptive statistics.

VI. Interest of research

In a context marked by high uncertainties in the financial markets, with ricochet speculation on rates, the risk measurement is of double interest for a Central Bank. Firstly, that of measuring its risk profile and secondly that of the efficient management of these foreign exchange reserves from different country of the Central African region, through the optimal allocation of these financial assets. This study will test the techniques of risk measure and optimal portfolio detection in order to assess their usefulness for a better precaution against market risk.

VII. Organization

This thesis is divided into two main parts, each containing two chapters: the first part deals with the conceptual and theoretical aspects of risk measure. We will begin this part by presenting the conceptual framework of risks in the first chapter. In this chapter, it is a question of defining the key concepts of market risk and presenting the coherent measures of risk. In the second, we will present the theoretical aspects on Conditional Value at Risk Portfolio Optimization.

In the second part of the work, we will present the analytical aspects relating to market risks, in particular interest rate risks. The third chapter provides a descriptive analysis of the asset management portfolio, using CVaR Portfolio Optimization to solve our problem. Finally, the fourth chapter will be devoted to investigating the problem from numerical data and discussing the results.

PART I: CONCEPTUAL AND THEORETICAL ASPECTS OF MARKET RISK ON THE ASSET MANAGEMENT PORTFOLIO

CHAPTER 1: UNDERSTAND THE MARKET RISK

In this chapter, we will present in a most explicit way the terms relating to market risk; how to measure it taking into account the extreme events of the market.

I. Definition

I.1 Market risk

In this study, the market risk we are referring to is the risk of loss of position of an asset, rate, currency or commodities. This risk is often due to adverse fluctuations in market parameters. These parameters are interest rates, exchange rates, stock prices, stock prices and stock indices. In controlling market risks, it is wise to analyze firstly the positions in terms of risk factors.

We define the factor as the origin of the risk. Broadly speaking, it is any item whose existence and / or future development is unknown and unpredictable today and which is likely to influence the market value of a given portfolio.

The risk factor can be of different types:

- **Financial risk :** from a sudden change in an economic parameter in the organization's environment;
- **Natural risk:** that is to say, resulting from the forces of nature;
- **Operational risk:** generally speaking, it is a risk that arises in the exercise of the profession. It is also apprehended as a risk of loss or fraud, failures in the procedures of the tasks to be performed;
- **Human factor:** that is to say, whose triggering is due to the action of man;

  The human factor can be:
  - ✓ Unintentional: this is an error in performing these functions, an omission;
  - ✓ Be voluntary: we distinguish:
    - the « smart little » type of modifying the operation of the system by omitting certain procedures in force so that other users are unaware of it.

- the malicious type: it is an intentional act of nuisance towards others.

Raoul Fokou in its research[5], says that the market risk of a portfolio comes from changes in the prices of financial assets and their effect on the total financial value of the portfolio. Thus, in terms of yield, the returns of assets then become random variables whose future evolution is not a priori known and quantifiable.

In general, there are two main categories of risk:

- **Specific risk:** this risk is inherent to the company's operating activity and its management, regardless of the influence of the market. It is indeed independent of the phenomena affecting the market as a whole. In terms of portfolio management, only systematic risk is remunerated by investors. The specific risk is, by definition, diversifiable by an optimization of the securities portfolio. It is therefore unpaid by investors in the market.
- **Systemic risk:** still called systematic risk, it corresponds to the incompressible risk attributed to the volatility of the market as a whole. Unlike specific risk, systematic risk is not diversifiable by optimizing a portfolio of securities and is therefore remunerated by investors in the market. This risk is considered irreducible and intrinsic to equity markets.

I.2 The components of market risk

The market risk itself is subdivided into several sub-funds: interest rate risk, currency risk, equity risk and risk on financial assets.

I.2.1 Interest rate risk

Interest rate risk is the risk of a change in the price of a debt security (bonds, negotiable debt securities) or a compound security or a derivative product resulting from a change in interest rates.

[5] *Raoul FOKOU*, *« **Measuring the Market Risk of an Equity-type Portfolio (Value-At-Risk, conditional Value-At-Risk)** », EURIA, 2006.*

The components of the interest rate

Any interest rate charged by a bank on these debts, or paid on its debts, consists of a number of components, some of which are more easily identifiable than others. In principle, a rate consists of five elements[6] :

- **The risk-free rate**: this is the fundamental component of an interest rate that represents the theoretical reference rate that an investor would expect from a risk-free investment at a given time.
- **The duration premium demanded by the market:** the price or valuation of a long-term instrument is more vulnerable to changes in market rates than in the case of a maturing instrument. To account for uncertainties surrounding both cash flow and prevailing interest rates, and the resulting price volatility, the market requires a premium or yield differential, relative to the benchmark rate of return in order to cover this duration risk**.**
- **The market liquidity premium**: even if the underlying instrument is risk free, the interest rate can incorporate a premium that represents the market's appetite for investment and the presence of buyers and sellers.
- **The general credit risk premium**: distinct from the yield spread for idiosyncratic credit risk, this premium represents the premium applied by market participants to a given credit quality (for example the additional return a debt instrument an issuer AA must produce in addition to a risk-free equivalent).
- **The idiosyncratic yield spread**: it reflects the specific credit risk associated with the signature of a given borrower (which also incorporates risk assessment from the sector, currency or location of the borrower) as well as to the debt instrument in question (different, for example, depending on whether it is a bond or a derivative).

In theory, these interest rate components apply to all credit risk exposures but, in practice, are more evident in instruments traded on the markets (such as bonds) than ordinary loans.

I.2.2 Currency risk

Currency risk is related to investments made in foreign currencies. In the case of a currency purchase transaction on Bloomberg, for example, when the dollar is rising, that is to

[6] *Basel Committee on Banking Supervision "**Interest rate risk in the banking portfolio**", 1997.*

say, appreciates against the euro, the institution risks losing any further the value of this transaction by converting the amount received in return for the sale of the dollars in euros when the dollar had declined. As the BEAC securities portfolio is valued in euros, it does not run any exchange risk because the national currency (the CFA Franc) used by the BEAC is in a fixed exchange rate with the euro.

I.2.3 Equity risk

An action is a title deed corresponding to a share of capital. On the other hand, it is important to know that this capital is subject to the risk of loss called equity risk. Equity risk is the risk that the holder of equities incurs in changing stock market prices.

I.2.4 Risk on financial assets

A financial asset is a security or contract that is generally transferable and tradable (for example in a financial market), which is likely to generate income or capital gains for the holder in return for a certain amount of risk taking.

The risk on financial assets is the risk to the holder of financial assets of the evolution of the price of these assets on the markets. This risk is amplified by the « *Mark to market* » accounting rule[7], which requires asset depreciation in the event of a fall in the value of the assets.

II. Risk measures

The risk is related to the volatility of the Mark to market (or market price valuation) of the asset portfolio. For a very long time, the natural measure of risk has been volatility. This is why in the Markowitz portfolio model, the agent maximizes his expectation of gain for a given risk level, which is measured by the standard deviation.

II.1 The Mean-Variance approach

Harry Markowitz was the founder of modern portfolio theory in 1952. Starting from the assumption that the risk of portfolio is correctly measured by the variance of its profitability,

[7] It is an asset valuation at the market price. Market to market refers to the day-to-day recording of the value of an asset according to its market price. The mark to market value corresponds to the market price of a portfolio.

Markowitz proposes as a measure of the risk associated with the return of an investment the standard deviation from the average of the distribution of returns. In the case of a portfolio of assets / liabilities, the risk is measured through the covariance of the different asset / liability peers, as:

$$Cov(X;Y) = E(X;Y) - E(X)E(Y)$$

where X and Y represent random returns.

The main innovation introduced by Markowitz is to represent the randomness of the return in another terms to measure the risk of an asset portfolio via the multivariate distribution of the returns of all the assets making up this portfolio.

Given a portfolio with N stocks, x_i shares invested in each stock ($i = 1, 2,\ldots,N$), for each stock, the price is given by p_1, p_2,…., p_n; so, the value of the market is given by:

$$V = \sum_{i=1}^{N} p_i x_i \,.$$

Then, the weight of the investment for each asset are given by:

$$w_i = \frac{p_i x_i}{V},$$

where we have : $$\sum_{i=1}^{N} w_i = 1.$$

Now, we can obtain the expected return of the portfolio as:

$$E(R_p) = \mu_p = \sum_{i=1}^{N} w_i \mu_i \,.$$

Noted that, the μ_p is the mean of the return of the portfolio. Therefore, it is used to calculate the performance of the portfolio.

In way to measure the risk of the portfolio, its variance can be employed:

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{i,j} \,.$$

Given the expected and the variance of the return of the portfolio, the investors have to separate the efficient portfolios to the inefficient portfolios based on the mean-variance criterion. The selection of the portfolio regarding the means and variances of their returns, the investors make the choice of the higher expected return for a given level of the variance or the lower variance for a given expected return[8].

Mean-variance criterion

Given two variables *X* and *Y*, with their means μ_X , μ_Y and variances σ_X^2 ,σ_Y^2. In order to say, respectively *X* is better to *Y* if and only if the following conditions hold together:

a) $\mu_X \geq \mu_Y$;

b) $\sigma_X^2 \leq \sigma_Y^2$;

c) at least one of two inequality holds in narrower sense.

All the efficient portfolios constitute the *efficient frontier*. To attain the efficient frontier, we have to find the portfolio that minimizes the risk for a given expected return, in another case a portfolio that maximizes the expected return for a given level of risk.

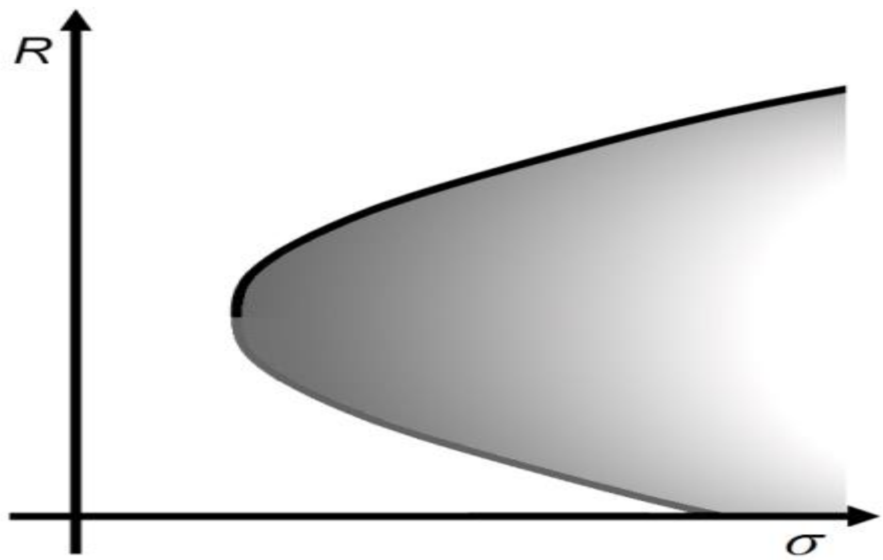

Figure 1: Efficient frontier.

[8] "*CITE, Mean-varaince criterion,* *https://financial-dictionary.thefreedictionary.com/Mean-variance+criterion saw 1/3/2018*

All possible portfolios are in the area shaded in gray. The upper part of the parabolic region (black line) is called the efficient frontier of risky assets and is formed by the portfolios which minimize the variance for a given expected return[9].

The basic formulation of Markowitz's portfolio selection problem is given by:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & x'\mathbf{V}x \\ \text{subject to} \quad & x'r = \pi \\ & x'e = 1 \end{aligned}$$

where

- x is an *N*- vector whose components $w_1,..., w_n$ denote the weight or proportion of the wealth allocated to the *i–th* asset in the portfolio;
- $\mathbf{V}$ is the $N \times N$ matrix of variances and covariances between assets[10];
- **r** is an *N*- vector of means with returns $r_1,..., r_n$ of *N* assets;
- π is the expected rate of return that investors whishes the portfolio to select realizes;
- e is the *N*- vector of ones.

However, in algebraic terms, this optimization problem can be rewritten such as:

$$\min(\sigma_p^2) = \sum_i \sum_j w_i w_j \sigma_{i,j}$$

$$\text{s.t. } R_p = \sum_{i=1}^{N} w_i r_i \quad \textit{and} \quad \sum_{i=1}^{N} w_i = 1$$

Markowitz's view of risk measuret has limitations; it can only be used as a measure of interdependence if the distributions are elliptical (distributions where the equity-density surfaces are ellipsoids)[11]. As a result, the Markowitz model is more suitable for elliptic

[9] *Valdemar Antonio Dallagnol Filho, « **Portfolio Management Using Value at Risk: A Comparison between Genetics Algorithms and Particle Swarm Optimization** », University Rotterdam, 2006,* *https://d1rkab7tlqy5f1.cloudfront.net/TBM/Over%20faculteit/Afdelingen/Engineering%20Systems%20and%20Services/People/Professors%20emeriti/Jan%20van%20den%20Berg/MasterPhdThesis/valdemar.pdf, pg.8.*

[10] Notice that $\mathbf{V}$ is nonsingular matrix. In way to verify this assumption, none of the asset returns must be perfectly correlated with the return of the portfolio and none of the assets is riskless.

[11] This is the case of the normal or the t-distribution with finite variances. A symmetric distribution is not necessary elliptic. The use of this framework with assets that present returns defined by non-elliptic distributions can seriously underestimate extremes events that may great losses (Szego, 2002).

distributions as the normal distribution with finite variances. However, empirical tests argue that even in the case of non-elliptic distributions, the variance-covariance model remains applicable under a single reserve, the underestimation of extreme events and the losses that can be associated with them (Kondor and Pafka (2001), Putmum et al (2002), Chan and Tan (2003)).

II.2 Beta approach

A few years later after the mean-variance measure, a new concept arose. This is the beta (**β**) that is the coefficient of the Capital Asset Pricing Model (CAPM). This measure tool compares the movements made by an asset relative to its reference market, which enables it to determine its level of risk relative to other reference assets. This measure is done by comparing the profitability of the assets with the market.

Mathematically, the beta of a financial asset is defined as the ratio of the covariance of the profitability of the asset with that of the market to the variance of the profitability of the market. Its formula is:

$$\beta = \frac{Cov(r_p, r_m)}{Var(r_m)}$$

with r_p the profitability of asset and r_m the profitability of the market.

The simplest way to calculate a Beta is the historical method. The historical profitability data from the asset to the market will be compared.

Concretely, we can cite the example of a CAC 40 action which is a stock index[12] that has a beta of 1.8: if the CAC 40 varies by 10%, the action should theoretically vary by 18%, it will amplify the movements of the market.

Conversely, if the stock has a beta of 0.7, it will vary less violently than its benchmark market: if the CAC 40 varies by 10%, it should move 7%.[13]

[12] Stock index: A key indicator for determining the performance of a market. Equity indices allow investors to manage their equity portfolios. They are representative of either a market is the case of the CAC 40, which measures the evolution of the 40 largest French capitalizations; a particular sector of activity such as the sectorial automotive index.

[13] *www.abcbourse.com Given an example on CAC 40 action to explain a beta measure*

Later, other studies found limitations in the calculation of beta. This will lead Ross to implement the Arbitrage pricing theory (APT) model that embodies arbitration valuation theory. This model is a generalization of the CAPM which uses not just one beta but a series of several beta coefficients, each of which corresponds to a particular factor of price and yield variation.

II.3 Value at Risk

In the 1990s, a new measure of risk appears: VaR (Value at Risk). Indeed, there were already limits to traditional risk measures.

- It assumes that returns are normally distributed which is not always the case. In general the distributions of returns of financial instruments present skewness /asymmetry and "fat tails"[14].
- Investors prefers portfolio with minimum variance for a given expected return for any quadratic function[15].
- The expected profitability of the portfolio is not always accurate to that of the market.

It was necessary to find measures that are more related to the overall distribution of money flows in a portfolio. It is in this context that VaR has been proposed. This measure was firstly used to quantify the market risk to which the bank portfolios are exposed. Indeed, the Basel Accord in 1997 required banks to hold regulatory capital to offset market risks calculated from VaR. This measure has become increasingly popular for assessing the risk of institutional or individual portfolios.

In general, Value at Risk is defined as the maximum potential loss that a portfolio may experience over a time horizon with a given probability threshold. Its determination depends essentially on three aspects:

- **the distribution of the returns of the portfolios**: more often this distribution is assumed normal but many financial actors use historical distributions. The difficulty

[14] Skewness is a term in statistics used to describe asymmetric in normal distribution in a set of statistical data. Skewness can come in the form negative skewness or positive skewness depending on whether data points are skewed to the left and negative or to the rights and positive of the data average. The dataset that shows the characteristic from a normal bell curve.

[15] S. Rachev, D. Martin, B. Racheva, and F. Stoyanov, "***Stable ETL optimal portfolio and extreme risk management***", Springer – Physika Verlag, 2007.

lies in the size of the historical sample: if it is too small, the probabilities of high losses are not very precise, and if it is too big, the concordance of the results is lost (comparisons of incomparable results are compared);

- **the chosen level of confidence (95% or 99% in general)**: the probability that portfolio or security losses will not exceed Value at Risk;
- **the time horizon**: this parameter is very important because the longer the horizon, the greater the losses can be. For example, for a normal distribution of returns, multiply the Value at Risk to one day by the square root of T, to have the Value at Risk in T days.

VaR models are models for estimating the degree of exposure of a portfolio to market risk, in order terms adverse changes in prices, interest rates, exchange rates and so on. These models may determine the potential maximum losses that a portfolio should suffer as a result of an unfavorable price movement of securities over a given horizon with a confidence level set.

For a given horizon and a probability level α such that $0 < \alpha < 1$, the VaR α designates the maximum expected loss during the predetermined period with a probability (1-α).

Value at Risk is no more than a fractile of the profit / loss distribution associated with holding an asset or portfolio of assets over a period of time. If we consider a coverage rate of α% (or equivalent a confidence level of 1-α%), the Value at Risk simply corresponds to the α% level fractile of the distribution of losses / profits valid on the holding period of the asset:

$$VaR(\alpha) = F^{-1}(\alpha)$$

where F (.) is the cumulative probability function associated with the distribution of losses / profits. Value at Risk is usually a loss (negative value). However, a risk value is often defined not from the (-) / profit (+) loss distribution, but from a profit (-) / loss (+) distribution. Such a definition aims to omit the minus sign before the loss and thus display only a positive VaR. In this case, the definition of VaR is the opposite of the loss / profit fractile.

$$VaR(\alpha) = -F^{-1}(\alpha)$$

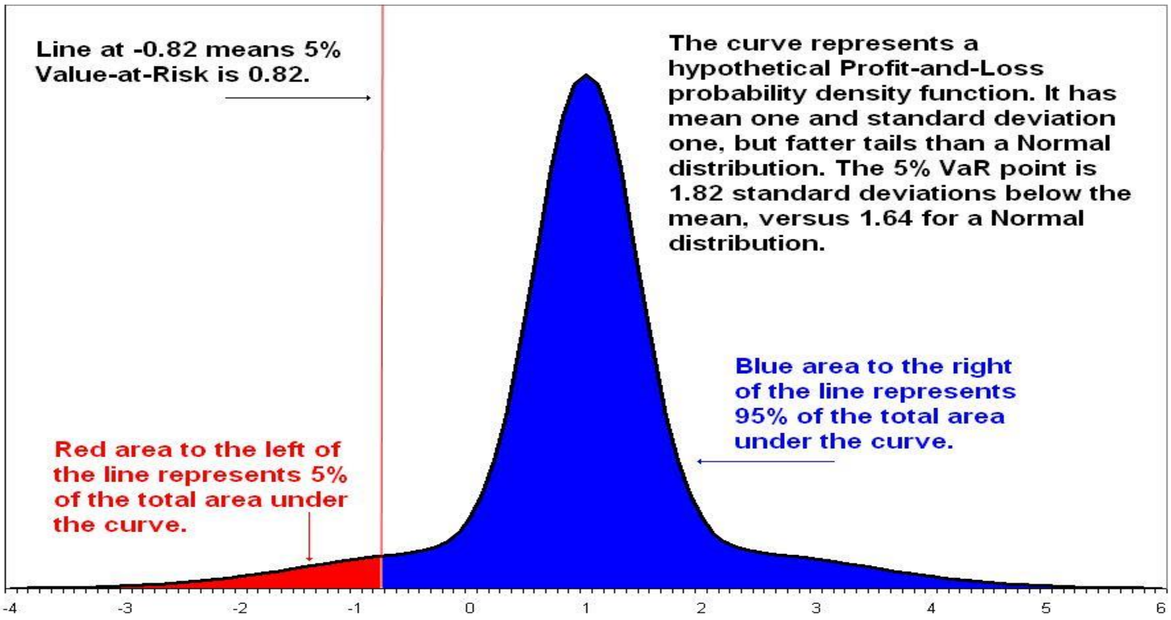

Figure 2 : Value at Risk Representation.

There are three methods for estimating VaR: the parametric method which assumes that portfolio returns follow a normal distribution, the non-parametric method or historical simulation method which does not impose a parametric distribution of losses and profits, and finally the Monte Carlo method which takes into account the occurrence of extreme events.

The parametric method or normal delta

Introduced by JP Morgan from October 1994 with the RiskMetrics system, the parametric method essentially derives its existence from the VaR measure based on the estimation of the variance / covariance matrix of the rates of return of the assets comprising the portfolio. The parametric VaR is determined by means of a relatively easy analytical calculation in practice but under rather restrictive theoretical assumptions. These assumptions assume that, on the one hand, the probability laws governing the distributions of changes in market prices are normal and, on the other hand, the instruments have a linear risk profile. Under these assumptions, the variance / covariance matrix can be applied relatively directly to the positions held to calculate the VaR.

In addition to the hypothesis of stationarity of returns valid for the methods in which the current distribution of risk factors is assumed to be identical to that of past factors and that the relationship between variation of portfolio and variation of risk factor is stable, the method parametric is based on two main assumptions:

- The hypothesis of normality

Based on the assumption that changes in market prices and rates are normally distributed, the rate of return on each of the financial assets is normally distributed if the valuation of the portfolio positions is a linear function of the movements of the factors' the market. Since the portfolio is a linear combination of these, its rate of return itself follows a normal pattern. In practice this assumption is not always verified which pushed JP Morgan to soften it by adding two other hypotheses:

- ✓ the volatility of returns varies over time;
- ✓ returns are self correlated.

- The assumption of linearity

In practice, the asset management portfolio of a bank often contains positions with non-linear characteristics, that is, whose sensitivity to changes in underlying prices and rates is not constant. These non-linear positions include, for instance, options and fixed income securities. In this case, to maintain the many benefits of normality, the so-called or RiskMetrics approach is implemented. It consists in using a linear approximation of the valuation of the portfolio market position.

The normal delta method therefore estimates the VaR of a call option in the following way. δ (the delta of the option) represents the proportion of the change in the value of the call which is explained by a price change of the sub-component. underlying. It is between 0 and 1. The normal delta method assumes that the delta is constant, that is to say that, whatever the price of the underlying, a variation in this price always has an identical impact on the value of the position.

In general, when the non-linearity of the position in question is very pronounced, a second risk factor, the gamma risk, which refers to the risk of variation of the delta, must be taken into account. The techniques used to calculate the VaR of such portfolios are the "delta-gamma methods".

Let $V(t)$ be the value of the portfolio at time t, we have:

$$V(t) = a^t F(t)$$

where $F(t)$ is the Gaussian vector of the law factors $N(\mu,\Sigma)$ and a the vector of sensitivity to these factors. In t, $V(t+1)$ is therefore a normal random variable of law $N(a^T\mu, a^T\Sigma a)$ [16]. The value of the VaR for a confidence threshold α then corresponds to:

$$\Pr((V(t+1)-V(t)) \geq -VaR_\alpha) = \alpha$$

we then obtain:

$$VaR_\alpha = \phi^{-1}(\alpha)\sqrt{a^T\Sigma a}$$

where $\Phi_\alpha^{-1}(\alpha)$ is the quantile at $\alpha\%$ of the reduced normal centered law and Σ the variance-covariance matrix.

The main basis for estimating parametric VaR is therefore the calculation of the variance-covariance matrix of portfolio returns.

The historical simulation method

This method is based on a history of the variations of risk factors at a given time horizon. This method is relatively simple but presents a measurement risk related to the choice of the sample. If this is too short, there is a risk that there will not be enough data to estimate the quantile at 99% (the variance of the estimator will be very large). If, on the contrary, it is chosen too long, there is the risk that the distribution of factors will change, which leads to a risk on the estimate of the quantile.

In practice, the different steps to calculate the sample VaR at the threshold of a portfolio at a date t with the historical method are as follows (the length of the history is determined in advance at N days, which we set for example at 3 years = 252 x 3 = 756 days):

- ✓ retrieve the composition (name of each asset and amount of assets) of the portfolio at the date t;
- ✓ calculate the N historical returns for each asset in the portfolio at time t;
- ✓ redial the historical distribution of portfolio values (with its composition at date t): calculate its notional value at the first historical date and apply the returns of each asset that compose it on each date up to the date t;

[16] *Antoine BEZAT & Ashkan NIKEGHBALI « **The theory of the extremes and the management of the market risks** », ENSAE, 2000.*

- ✓ classify and number in ascending order the N different variations (losses or gains) fictitious portfolio reconstituted and thus obtain a distribution of N variations;
- ✓ VaR at threshold α (for example 99%) which represents the maximum loss that the portfolio will realize in 99% of cases (α * N).

Monte Carlo simulation methods

The Monte Carlo simulation method uses an econometric model to determine the evolution of risk factors over time, the parameters of this model being set by the user or estimated from past data. Whereas the Historical Simulation method is based on the only scenario based on the past behavior of the markets, the MC simulation method is based on a multitude of random scenarios defined from the econometric model. MC's simulation method is flexible in terms of modeling and allows for the treatment of all market positions once the price behavior of the different products in the portfolio has been modeled. However, it is complex and difficult to implement, not to mention its high cost, especially in computing time.

III. Coherent Risk Measures

Consider by ρ a risk measure, we denote by A_ρ a relation between a space *X* of random variables and a non-negative real number *R*:

$$A_\rho = \{X \in R / \rho(X) \geq 0\}.$$

This acceptance set associated, indoor to be considered a risk measure must satisfy some properties[17]:

Translation invariance: $\rho(X + r_0) = \rho(X) - \alpha$, for all random variables *X*, real numbers α and all riskless rates r_0 on a reference instrument. It means that adding the riskless rate r_0 and (respectively subtracting α), imply to decrease (respectively increase) the risk by α .

Subadditivity: $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$, for all random variables X_1 and X_2. That can be inperted by "merger does not create extra risk measure". Otherwise, in order to decrease risk, it could be convenient to spit up a company into different distinct divisions. It can be

17 *Artzner et al*, ***Coherent Measures of Risk***, *July 22, Université Louis Pasteur, Eidgenossische Technische Hochschule, Société Générale, Carnegie Mellon University*, *1998*

proved that any positively homogeneous functional ρ , is convex if and only it is subadditive[18] (**Szego, 2005**).

Positive homogeneity: $\rho(\lambda X) = \lambda\rho(X)$, for all random variables X and real numbers $\lambda \geq 0$. The position size should influence risk then the consequences of lack of liquidity when computing a future value of a position is considered.

Monotonicity: $X_1 \leq X_2$ implies $\rho(X_1) \leq \rho(X_2)$, for all random variables X_1 and X_2.

Relevance: $X \leq 0$ and $X \neq 0$ implies $\rho(X) > 0$, for all random variable X.

Regarding the set of properties above, all risk measures which we defined before in the previous section even VaR could not consider as a coherent risk measure. Only, in the special case which the join distribution of returns is elliptic, VaR is subadditive (**Embrechts, 2000**).

Conclusion

We note that there are several types of market risk. Taken as the loss of position of an asset as a result of a change in market conditions. It is characterized by two risk categories: the systemic risk that corresponds to the incompressible risk associated with the entire market is the specific risk that is independent of the phenomena that affect the market as a whole.

Market risk measure techniques began with Markowitz. The first techniques consisted of calculating the spreads between the distributions of the returns of the modern portfolio in order to measure the level of return of the portfolio. These techniques evolved into the Beta model of the financial asset valuation model, which would measure the profitability of assets to that of the market. A few years later, the Basel Accord introduced VaR as a regulatory risk measure for all financial institutions. All these aforementioned risk measures have shown their limit over time. This will lead to the specification of consistent risk measures.

[18] *Giorgio Szego, **Measure of risk**, European Journal of Operational Research, 2005.*

CHAPITER 2: Conditional Value at Risk Portfolio OPTIMIZATION

Before addressing the estimation of the CVaR for our portfolio, we will first make a historical review on the subject and define all steps.

I. Conditional Value at Risk

Several researches have been done to develop the risk measures. Most of these researches focus on extreme events in the tail distribution where the portfolio loss occurs (variance does not differentiate loss or gain), and quantile-based models have thus far become the most popular choice[19]. As we say before, Conditional Value at Risk (CVaR) developed by Rockafellar and Uryasev, also known as Expected Shortfall[20] by Acerbi and Tasche (2001). On the theoretical view, Artzner et al (1997, 1999) mentioned that CVaR is a coherent risk as we define before. On the partical view, the convex representation of CVaR from Rockafellar and Uryasev opened the door for convex optimization and gave it large advantage in implementation (Jing Li and Mingxin Xu, 2013).

Several research papers present the Conditional Value at Risk (CVaR) as a coherent risk measure. CVaR is the expected loss that we are in the *q*% left tail of the distribution.

Consider by $L(x, y)$ the loss function, where *x* is the decision vector choose from some set $X \subseteq R^n$. We can interpret *x* as a portfolio with *X* a set of available portfolios and y the market prices. For each *x*, the loss function $L(x, y)$ can be seen as a random variable which is characterized by *p(y)* denoted as the probability distribution of *y*. Specified by $x = (x_1, \cdots, x_n)$ the portfolio of assets for which, x_i represents the position for each instrument *i* in the portfolio following that:

$$x_i \geq 0 \quad \text{for} \quad i = 1, \cdots, n \quad \text{with} \quad \sum_{i=1}^{n} x_i = 1$$

[19] *Jing Li and Mingxin Xu, "**Optimal Dynamic Portfolio with mean-CVaR Criterion**", Risks, 2013, pp. 2-3.*

[20] In case of continuous random variables, the definition of Expected Shortfall coincides with that of CVaR. (Szgo, 2005).

However if *p(y)* is continuous, the probability density function of $L(x,y)$ is continuous; in order to simplify, the methods for minimization (**Uryasev, 1995).**

The probability that $L(x,y)$ does exceed the threshold α , is given by (**Qin et al, 2014**):

$$\psi(x,\alpha) = \int_{L(x,y)\le\alpha} p(y)dy \cdot$$

As a function of α for fixed x, $\psi(x,\alpha)$ is the cumulative distribution function for the loss associated with x. It completely determines the behavior of this random variable and is fundamental in defining VaR and CVaR (**Krokhmal et al, 2001**).

The VaR of the loss assocaited with and a specifial probability level β in (0,1) is the value:

$$VaR_\beta(x) = \min\{\alpha \in R^m : \psi(x,\alpha) \ge \beta\}.$$

Compared to VaR, CVaR has some superior mathematical properties[21].

$$\begin{aligned} CVaR_\beta(x) &= E[L(x,y)/L(x,y) \ge VaR_\beta(x)] \\ &= (1-\beta)^{-1} \int_{L(x,y)\ge VaR_\beta} L(x,y)p(y)dy \end{aligned}$$

The $\beta - VaR$ value for the loss random variable associated with *x* comes out as the left endpoint of the nonempty interval[22] consisting of the values of α such as $\psi(x,\alpha) = \beta$. In the $\beta - CVaR$ value for the loss random variable associated with *x*, the probability that $L(x,y) \ge VaR_\beta(x)$ is therefore equal to $1-\beta$. Thus, $CVaR_\beta(x)$ comes out as the conditional expectation of the loss associated with *x* relative to that loss being $VaR_\beta(x)$ or greater.

The key to the approach is a characterization of $CVaR_\beta(x)$ and $VaR_\beta(x)$ in terms of the function F_β on X × R that is defined by

$$F_\beta(x,\alpha) = \alpha + (1-\beta)^{-1}\int_{y\in R^n}[L(x,y)-\alpha]^+ p(y)dy,$$

---

[21] *Rockafellar, R.T., Uryasev, S.: **Optimization of conditional value-at-risk**. Journal of Risk 2(3), pages 21-41, 2000.*

[22] This follows from $\psi(x,\alpha)$ being continuous and nondecreasing, the interval might contain more than single point if ψ has « flat sopts » (Krokhmal et al, 2001).

where α^+ is define as $\max(\alpha,0)$.

For instance, CVaR can be obtained by the following equation

$$CVaR_\beta(x) = \min_{\alpha \in R} F_\beta(x,\alpha).$$

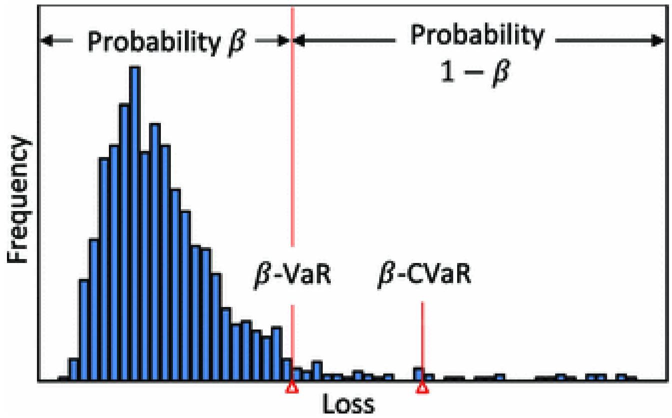

Figure 3: Conditional Value at Risk Representation

A linear programming techniques is demonstrated by Rockafellar and Uryasev (2000), can be used for optimization of the Conditional Value-at-Risk (CVaR) risk measure. A simple description of the approach for minimizing CVaR and optimization problems with CVaR constraints can be found in (**Uryasev, 2000**). Most experimental studies showed that risk optimization with the CVaR and constraints can be done for large portfolios and a large number of scenarios with relatively small computational resources. A case study on the hedging of a portfolio of options using the CVaR minimization technique is included in (**Rockafellar and Uryasev, 2000**). Mauser and Rosen (1991) was the first to study this problem with the minimum expected regret approach. Also, the CVaR minimization approach was applied to credit risk management of a portfolio of bonds, Andersson at al. (1999)[23].

[23] *Pavlo Krokhmal, Jonas Palmquist and Stanislav Uryasev, "**Portfolio Optimization with Conditional Value at Risk Objective and Constraints**", Center for Applied Optimization, Dept. of Industrial and Systems of Engineering, University of Florida, Gainesville, FL 32611, 2001.*

II. Portfolio Optimization with CVaR constraints

In this section, we present an optimization model of maximizing expected returns subject to CVaR constraints. The key of this model is the function $F_{\beta}(x,\alpha)$ define in the previous section.

The main workflow for CVaR portfolio optimization is to create an instance of a Portfolio CVaR object that completely specifies a portfolio optimization problem and to operate on the Portfolio CVaR object using function whose we describe before to obtain and analyze efficient portfolios. A CVaR optimization problem is completely specified with the following four elements:

- A universe of assets with scenarios of asset total returns for a period of interest, where scenarios comprise a collection of samples from the underlying probability distribution for asset total returns. This collection must be sufficiently large for asymptotic convergence of sample statistics. Asset return moments and related statistics are derived exclusively from the scenarios.
- A feasible set that specifies the set of portfolio choices in terms of a collection of constraints.
- A model for portfolio return and risk proxies, which, for CVaR optimization, is either the gross or net mean of portfolio returns and the conditional value-at-risk of portfolio returns.
- A probability level that specifies the probability that a loss is less than or equal to the Value-at-Risk. Typical values are 0.9 and 0.95, which indicate 10% and 5% loss probabilities.

The simplest CVaR portfolio optimization problem has:

- Scenarios of asset total returns.
- A requirement that all portfolios have nonnegative weights that sum to 1 (the summation constraint is known as a budget constraint).
- Built-in models for portfolio return and risk proxies that use scenarios of asset total returns.
- A probability level of 0.95.

Scenario generation process

We approximate the integral in the CVaR function by the sum over all scenarios. This approach assume also a joint distribution for the price-returns process for all instruments. The process of scenarios of asset total returns allows for using historical data without assuming a particular distribution. We start to generate the scenarios with historic time series of prices for *n* instruments. We divide these series into *J* periods (scenarios), we can calculate the return over each of these periods. Over the same length of the period, we optimize the portfolio. For instance, minimizing over a one day period, we take the close prices of two consecutive days, p_t and p_{t+1} (**Uryasev, 1995**).

III. Proposed portfolio selection model

We propose here our portfolio selection model based on the one of **Qin et al, 2014**. Assume that, we have *n* risky assets in our financial market for trading. The investor would like to allocate the initial wealth V_0 . Let us introduce a few notation:

- r_i : the return of risky asset *i*;
- R_p : the total return of the portfolio;
- p_i : the price for each risky asset *i* each round lot;
- x_i : the position for each risky asset *i* invested;
- σ_i : an upper bound on risky asset *i*;
- ε_i : a lower bound on risky asset *i*;
- λ : the acceptable return for the portfolio in case the interest rate increase to 1%;

The total return of the portfolio can be rewritten as:

$$R_p = \sum_{i=1}^{n} x_i r_i$$

The intention of the proposed model is to minimize the CVaR in the case of the return of the portfolio is equal or greater than λ .

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \text{CVaR} \\ \text{subject to} \quad & \varepsilon_i \le x_i \le \sigma_i \qquad i = 1,2,\dots, n \\ & R_p / V_0 \ge \lambda \\ & \sum_{i=1}^{n} x_i p_i \le V_0 \end{aligned}$$

Now for simplicity, we consider that $V_0 = 1$, so our model can be rewrite as:

$$\begin{aligned} &\underset{x}{\text{minimize}} && \text{CVaR} \\ &\text{subject to} && \varepsilon_i \le x_i \le \sigma_i \\ & && -\sum_{i=1}^{n} x_i r_i \le -\lambda \\ & && \sum_{i=1}^{n} x_i = 1 \end{aligned}$$

This model consist to explore the constraints on the expected portfolio return, and upper-lower bound for the stock weights. In order to implement this model, we need to construct matrices of constraints: *A*, *Aeq*, *b*, *beq*, *lb* and *ub*.

The matrix *A* will give by:

$$A = \begin{pmatrix} -r_1 \\ -r_2 \\ \vdots \\ -r_N \end{pmatrix}$$

where $r_1, r_2, \cdots, r_N$ represent the set of stock average returns which are followed by the negative of an $N \times N$ identity matrix and an $N \times N$ identity matrix bellow. The two identity matrix represents the coefficients of weights in order to apply the upper bound and the lower bound for the portfolio weights.

And we have:

$$Aeq = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix},$$

$$b = -\lambda,$$

$$beq = 1,$$

$$lb = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{pmatrix},$$

and

$$ub = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_N \end{pmatrix}.$$

where *Aeq* is an $N\times 1$ column vector of ones, *beq* equals to 1, *lb* a lower bound and *ub* an upper bound for each *N* stocks weights. Now, we can minimize the CVaR function which the following constraints where *w* is an optimal weights:

$$A \cdot w \leq b \text{ and } Aeq \cdot w = beq.$$

Given the scenarios of asset returns, our problem can be solve by the Matlab toolbox which the code is presented to the Appendix 4.

PART II: ANALYTICAL ASPECTS OF MARKET RISKS LINKED TO THE BEAC ASSET MANAGEMENT PORTFOLIO AN APPROACH BY CVaR PORTFOLIO OPTIMIZATION

CHAPITER 3: DATA AND EXPLORATION OF CENTRAL BANK'S PORTFOLIO

This chapter aims to describe in detail the portfolio used to answer our problem.

I. Source of data and normality hypotheses

I.1 Data

To calculate the CVaR from the techniques of Portfolio Optimization, we collected data on two years over the period from January 2015 and January 2017. Noting that during this period the portfolio was subjected to risk on the financial market. This is the temporal data taken from day to day on Bloomberg for 30 portfolio stocks assets BEAC. Over a year, we have daily data spread over 12 months and distributed over 22 working days of the month and consequently 252 working days of the year.

Normality Test

The first works in finance made very strong assumptions about the returns of financial assets. One of these assumptions is the normality of log-returns. So we perform here the normality test of log-returns' distribution.

Regarding the apppendix 2, we have the results of the Shapiro-Wilk, the Anderson-Darling, the Lilliefors and the Jarque-Bera tests. Before giving an interpretation of these results, we notice that all these tests give the same results.

Interpretation:

H_0: Securities returns follow a Normal law.

H_1: Securities returns do not follow a Normal law.

Since the calculated p-value is lower than the significance level $\alpha = 0.99$, the null hypothesis H_0 must be rejected, and the null hypothesis H_1 must be accepted.

In the simplest way, with a level of significance $\alpha = 0.99$ and with regard to the results of the tests, we note that the p-values of the tests are each equal to 0.0001 and are therefore below our threshold, therefore the distribution of the returns of the securities does not follow a normal law.

I.2 Presentation of the asset management portfolio

The asset management portfolio consists of stock assets and is managed passively.

BEAC PORTFOLIO		
DBR4	FRTR6	EFSF2
FRTR3	FRTR7	EFSF3
FRTR4	FRTR8	EFSF4
DBR3	CADES1	EIB1
FRTR2	CADES2	EIB2
FRTR1	CADES4	BGB1
DBR2	CADES3	CADES6
FRTR5	CADES5	CADES7
DBR1	SAGESS1	FRTR9
DBR5	EFSF1	EIB3

Table 1 : Asset management portfolio of BEAC

The asset portfolio allows to the Bank to generate fixed revenues based on rates of returns. In the event the rate cut, when the market value of portfolio decrease, the returns of the portfolio is negatively impacted. If the interest rate rise, the security's value is negatively impacted.

Based on the observed daily closing prices, we calculate the daily returns such as:

$$r_{i,t} = \frac{P_{i,t+1} - P_{i,t}}{P_{i,t}} \approx \ln\left(\frac{P_{i,t}}{P_{i,t+1}}\right)$$

and we estimate $\hat{r}_i$ by using the historical data:

$$\hat{r}_i = \frac{1}{T}\sum_{t=1}^{T} r_{i,t}$$

We refer to Appendix 3 for the Matlab code related to the calculation of the matrix of logarithmic returns.

II. Structure of the asset management portfolio of BEAC

The asset management portfolio of BEAC has 30 stocks assets issues to its credit. These are generally German, Belgian, Spanish, French, Dutch and other securities.

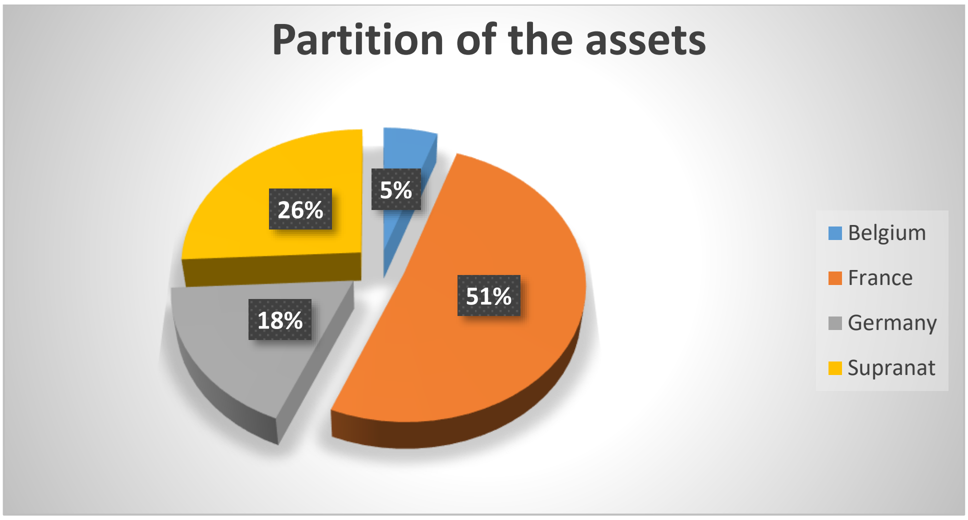

Figure 4: Breakdown of origins of securities in the portfolio

We note that most of the securities making up the portfolio are of French origin. French securities then represent 51% of total portfolio securities.

The figure of the evolution of the returns of all assets observed in the appendix 1 shows us that the returns of each title vary very little in the time. This evolution of return for each asset is significantly related to that of another and vice versa. This for all assets in the portfolio. In order term, these assets are strongly correlated. However, we can observe that the fluctuation of the returns of some assets is more dense than that of the others.

II.1 Performance of portfolio

Figure 5 : Evolution of overall portfolio performance

Figure 5 shows the evolution of the portfolio's total return over the study period. On the upper part of this figure, we observe that portfolio returns from January to April 2015 were up and then dropped to its lowest level of -2.019% as of June 9, 2015. After this date, yields returned on the rise to reach their highest level of 6.214% on September 8, 2016.

In the lower part we can observe in green the periods for which the total return is positive and in red when it is negative.

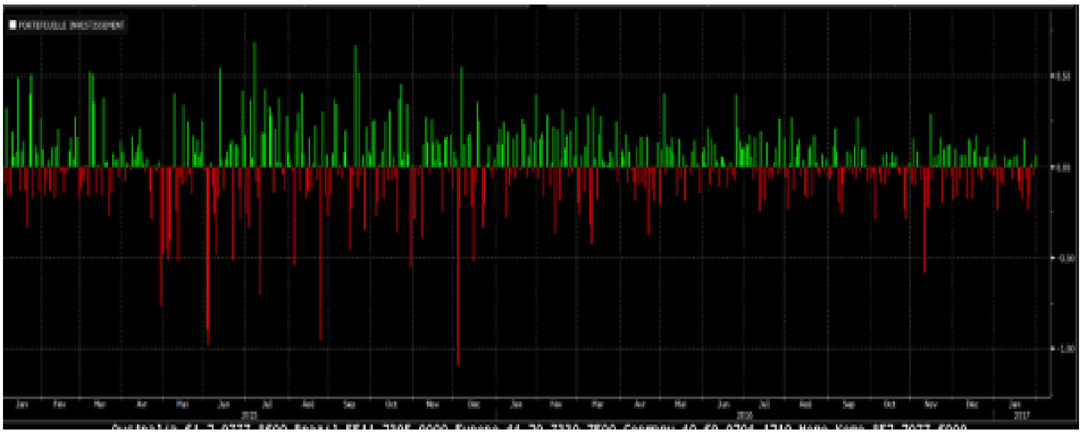

Figure 6 : Analysis of the overall performance of the portfolio over the period

In Figure 6, as in the previous one, we have positive trends in green and negative trends in red to determine portfolio performance. The three best performances of the portfolio were recorded on the following dates: 7 July, 18 September and 7 December 2015 and are estimated respectively as follows: 0.68%, 0.67% and 0.55%. The portfolio also recorded the worst performances on the following dates: December 3, June 3, and August 25, 2015, of -1.09%, -0.98%, -0.96%.

	Up	Down	Total
Number	294	248	542
Percentage	54.24	45.76	100
Mean	0.14	-0.16	0.01
Standard deviation	0.13	0.17	0.21
Sequence max	9	8	9
Sequance mean	2.03	1.71	1.87

	Performance	Date
Best 1	0.68	7/7/2015
Best 2	0.67	9/18/2015
Best 3	0.55	12/7/2015
Worse 1	-1.09	12/3/2015
Worse 2	-0.98	6/3/2015
Worse 3	-0.96	8/25/2015

Table 2: Total returns' performance on period

Figure 7 : Seasonal analysis of total portfolio return

In Figure 7, we perform a seasonal descriptive analysis of portfolio returns. In white, we have the histogram of the returns of the portfolio in 2015, in yellow that of 2016 and in red that of Jannuary 2017. We note that over the entire study period that is to say from January 2015 to January 2017, the portfolio posted its lowest return in June 2015 of -2.16%. Its highest return was recorded in January 2016 of 1.85%.

Years	Jan.	Feb.	Mar.	April	May	June	July	August	Sept.	Oct.	Nov.	Dec.
2015	N.A.	0.3	0.87	-0.95	-0.88	-2.16	1.59	-0.87	1.14	0.79	0.56	-1.05
2016	1.85	1.21	0.11	-0.69	-0.86	1.24	0.56	0	0.18	-0.97	-0.58	0.11
2017	-0.87	N.A	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.
Mean	0.49	0.75	0.49	-0.82	-0.01	-0.46	1.08	-0.43	0.66	-0.09	-0.01	-0.47

Table 3: Brief description of portfolio monthly return

II.2 Few performances for most important assets in the portfolio

Assets	Tot Ret 1D	Tot Ret MTD	Tot Ret YTD
SAGESS1	0.15	-1.27	-1.27
BGB1	0.06	-1.6	-1.6
CADES2	0.11	-1.5	-1.5
CADES3	0.13	-1.88	-1.88
DBR1	0.05	-0.22	-0.22
DBR2	0.06	-0.19	-0.19
DBR3	0	-0.12	-0.12
DBR4	0.01	-0.13	-0.13
DBR5	0.01	-0.12	-0.12
EFSF1	0.02	-0.02	-0.82
EIB1	0.02	-0.7	-0.7
EIB2	0.01	-0.83	-0.83
FRTR1	0.11	-2.25	-2.25

FRTR2	0.13	-0.66	-0.66
FRTR3	0.01	-0.09	-0.09
FRTR4	0.12	-0.88	-0.88
FRTR5	0.14	-0.57	-0.57
FRTR6	0.06	-0.23	-0.23
FRTR7	0.01	-0.2	-0.2
FRTR8	0.08	-0.3	-0.3
Global portfolio	0.06	-0.87	-0.87

Table 4: Performance of most dynamics assets

Total Ret 1D is the total 1-day return in the base currency of the portfolio at the time of the analysis. At the security level, this corresponds to the security's return only on the days it was held in the portfolio.

Total Ret MTD represents the total return since the beginning of the month in the base currency of the portfolio. At the security level, this is the return on the security only for the days it was held in the portfolio.

Ret YTD total is the total return since the beginning of the year in the base currency of the portfolio. At the security level, this is the return on the security only for the days it was held in the portfolio.

III. Statistics of portfolio

	3 months	6 months	1 year	2 years
Return				
Total return	-1.33	-2.12	-0.87	2.27
Maximum return	0.29	0.29	0.16	0.68
Minimum return	-0.59	-0.59	-0.24	-1.09
Average return (annualized)	-7.12	-5.73	-13.49	1.66
Average excess return (annualized)	9.11	7.26	23.46	0.22
Risk				
Standard deviation (annualized)	2.21	2.07	1.72	3.38
Risk of downside (annualized)	1.72	1.57	1.28	2.58
Asymmetry	-1	-0.69	-0.3	-0.9
VaR 95% (ex-post)	-0.23	-0.24	-0.24	-0.32
Tracking Error (annualized)	3.5	2.84	3.13	2.29
Return/Risk				
Sharpe Ratio	-2.18	-1.85	-5.53	0.4
Jensen Alpha	-0.85	-0.3	-1.8	0.58
Information Ratio	1.84	1.81	5.19	0.07
Measure of Treynor	-0.13	-0.09	-0.27	0.02

Beta (ex-post)	0.38	0.41	0.35	0.65
Correlation	0.9305	0.9373	0.9643	0.8729

Table 5 : Some key performance indicators for the portfolio

In Table 4, we have some temporal statistics (quarterly, half-yearly and annual) of the portfolio.

We illustrate here the vulnerability to risks of financial assets based on the uncertainty that reigns in the market in terms of information asymmetry between investors. It seeks to show, using the concepts developed by the equilibrium theory in rational expectations, that the price of a financial security can make it possible to solve the asymmetries of information existing between the agents having access to privileged information on the title value (insiders) and other agents (uninformed agents). An agent receiving favorable information about the value of an asset, seeks to take advantage of this information by placing purchase orders. This increase in demand causes an increase in price, which signals its privileged information. If uninformed agents correctly anticipate the relationship that may exist between price and insider information, they can make inferences about private information for each price level (**Grossman, 1976**). In view of the results in the table 4, the bank's agents always receive information opposite to the price of the securities on the market which increases the risk-taking.

The tracking error or active risk is also presented in finance as a risk measure in an investment portfolio that is due to active management decisions made by the portfolio manager; it indicates how closely a portfolio follows the index to which it is benchmarked. Consider the portfolio on 2 years, an risk of 2.29% would mean that approximately 2/3 of the portfolio's active returns (one standard deviation from mean) can be expected to fall between +2.29 and -2.29 per cent of the mean excess return.

The Sharpe Ratio measures the difference in the profitability of a portfolio of financial assets (e.g. equities) relative to the rate of return of a risk-free investment (i.e. the risk premium, positive or negative), divided by a risk indicator, the standard deviation of the profitability of this portfolio, in other words its volatility. For simplicity, it is an indicator of the (marginal) profitability obtained per unit of risk taken in this management.

If the ratio is negative, the premium for risk is negative and the situation is bad: the portfolio has a lower performance than a risk-free investment. We can observe this on the first three periods describe in table 4.

If the ratio is between 0 and 1, the over performance of the portfolio in relation to the benchmark is for risk taking too high. Or, the risk taken is too high for the return obtained. We notice this kind of results if we consider the portfolio on 2 years.

If the ratio is greater than 1, the return on the portfolio outperforms the benchmark for an ad hoc risk taking. In other words, the over-performance is not done at the cost of too much risk.

The Jensen's alpha is the risk-adjusted performance measure that represents the average return on a portfolio or investment, above below that predicted by the capital asset pricing model (CAPM), given the portfolio's or investment's beta and the average market return. Jensen's measure is one of the ways to determine if a portfolio is earning the proper return for its level of risk. If the value is positive, then the portfolio is earning the excess returns. Regarding the results we have, on 2 years this portfolio earns the excess returns.

The information ratio is a ratio that is equal to the average return of an asset relative to the average of a benchmark, divided by the tracking error of the benchmark. It is therefore a indicator of the effectiveness of the return/risk ratio associated with portfolio management, which is used to determine the extent to which an asset is performing better than a benchmark. Thus, a high information ratio means that the asset regularly exceeds the benchmark. The higher the information ratio, the better the portfolio management performs. More performance against the benchmark with less risk. The last quartile of investors and financial managers generally have an information ratio of 1.5.

The measure of Treynor, also known as the reward-to-volatility ratio, is a metric for returns that exceed those that might have been gained on a risk-less investment, per each unit of market risk. The Treynor ratio, developed by Jack Treynor, is calculated as follows: (Average Return of a Portfolio – Average Return of the Risk-Free Rate)/Beta of the portfolio. The Treynor ratio shares similarities with the Sharpe ratio. The difference between the two metrics is that the Treynor ratio utilizes beta, or market risk, to measure volatility instead of using total risk (standard deviation).

Ultimately, this measure attempts how successful an investment is in providing investors compensation, with consideration for the investment's inherent level of risk. The Treynor ratio is reliant upon beta that is, the sensitivity of an investment to movements in the market to judge risk. The Treynor ratio is based on the premise that risk inherent to the entire market (as represented by beta) must be penalized, because diversification will not remove it. When the value of the Treynor ratio is high, it is an indication that an investor has generated high returns on each of the market risks he has taken. Noting that in our case, the treynor ratio is lower for all periods. So, we have an inverse situation which the investor has generated lower returns considering the risk taking.

The Treynor ratio allows for an understanding of how each investment within a portfolio is performing. It also gives the investor an idea of how efficiently capital is being used.

The correlation is a statistic that measures the degree to which two securities move to each other. This value must fall between -1 and 1. The negative correlation means that there is lower relationship between the assets. A zero correlation implies no relationship to all assets. The positive correlation means that the assets are more related one to other. This is the case in this portfolio regarding the results in table 4.

CHAPITER 4: COMPUTATION OF MODEL AND DISCUSSION

In this chapter, we will discuss applying the techniques of CVaR Portfolio Optimization for our model using the data we have.

I. Optimal CVaR portfolio

As one of the mains of this study, we perform the CVaR Portfolio Optimization. Most institutional investors tend to use the CVaR to control downside risk and increase asymmetry, but their goal is not to deviate too much from Markowitz MV portfolios. To achieve this goal, Krokhmal et al. (2002) suggest using CVaR constraints to improve MV portfolio asymmetry. This method will be extended to the asymmetry of the Markowitz MV portfolios by adding one or more CVaR type constraints. The use of several CVaR-type constraints with confidence level β attempts to modify the distribution of returns according to expectations in terms of loss. For example, in the case of deadlines, we set the average values of the worst losses at 1% to be limited by certain values.

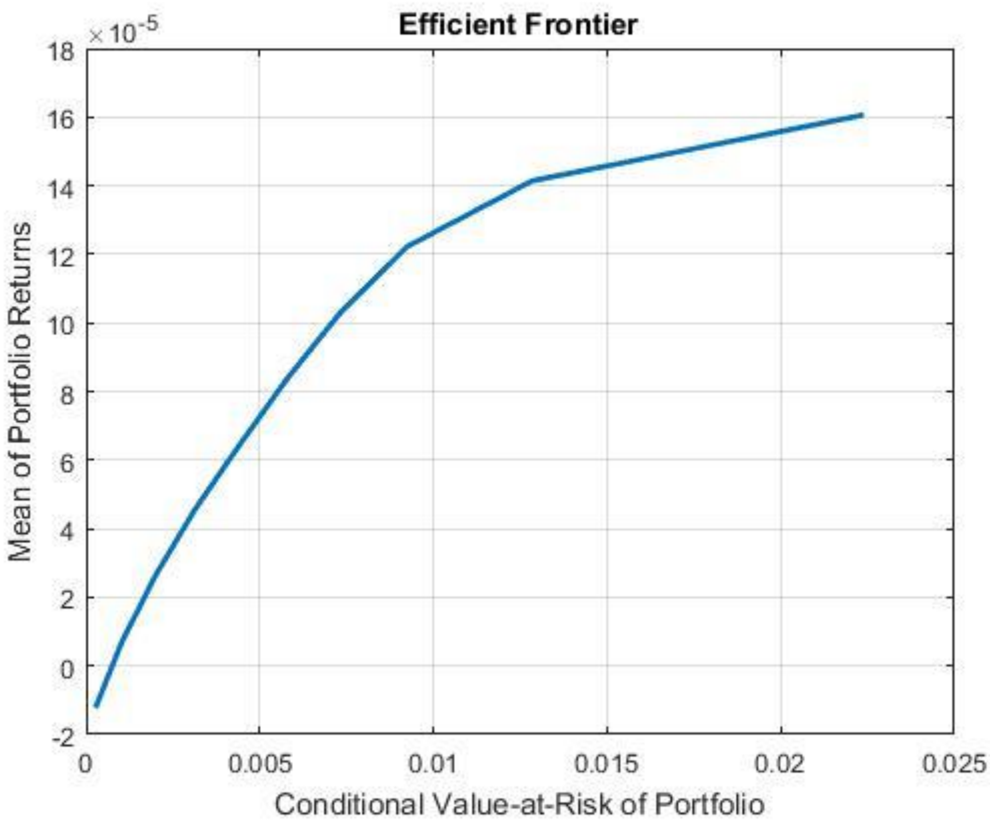

Figure 8 : Efficient frontiers of CVaR Optimal Portfolios

This figure shows, the efficient frontier of one period model with CVaR constraints of 30 assets from the portfolio constituted only of risky assets. We plot this graph of efficient frontier for β-value 0.95. The CVaR optimal portfolio was obtained by maximizing the expected returns subject to the constraint on the portfolio. Gradually as the CVaR scales for risk confidence level β = 95% increase, the whole efficient frontier moves rightward.

II. Optimal Mean-Variance portfolio

In this section, we decided to perform the Mean-Variance portfolio, in order to look at the portfolio problem according two different risk measures.

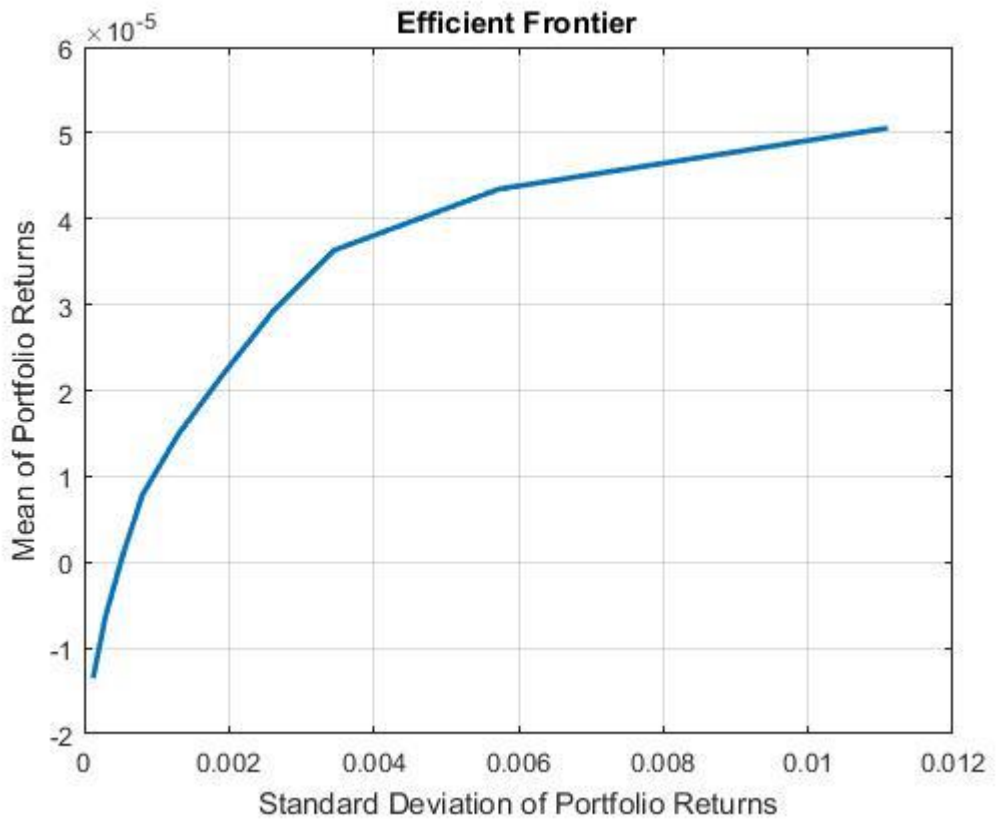

Figure 9 : Efficient frontier of Mean-Variance Optimal Portfolios

In the figure 9, we can observe the efficient frontier for Return / MV efficient portfolios. For each return on the vertical scale, it displays the standard deviation of the optimal portfolio CVaR with the confidence level β = 95% on the horizontal scale.

The difference between the MV and CVaR approaches is not very significant. Relatively tight graphs of optimal CVaR and MV portfolios indicate that an optimal CVaR portfolio is "near optimal" in MV, and vice versa, an optimal MV portfolio is "near optimal" in CVaR. This agreement between the two solutions, however, should not say that the portfolio management methods discussed are "the same". The results obtained are specific to the data set and the proximity of CVaR and MV optimization problem solutions to seemingly "close to normal" distributions (Mausser and Rosen, 1999, Larsen et al., 2002). These results thus confirm the hypothesis that the distribution of historical data used in our study will be normal. However, a normal distribution of returns data may not be a necessary assumption for a CVaR portfolio but this helps to explain the difference with a Mean-Variance portfolio which assume that returns are normally distributed.

III. Compare Mean-Variance with CVaR portfolios

Another way to compare an efficient portfolios is to look at the weights for the portfolios. We'll visualize the weights for the CVaR and the Mean-Variance side-by-side. This allows to see the mix of the different assets in each of the efficient portfolios. Let us take a closer look of the weights on the efficient frontier in case of the Mean-Variance and the CVaR portfolio optimization.

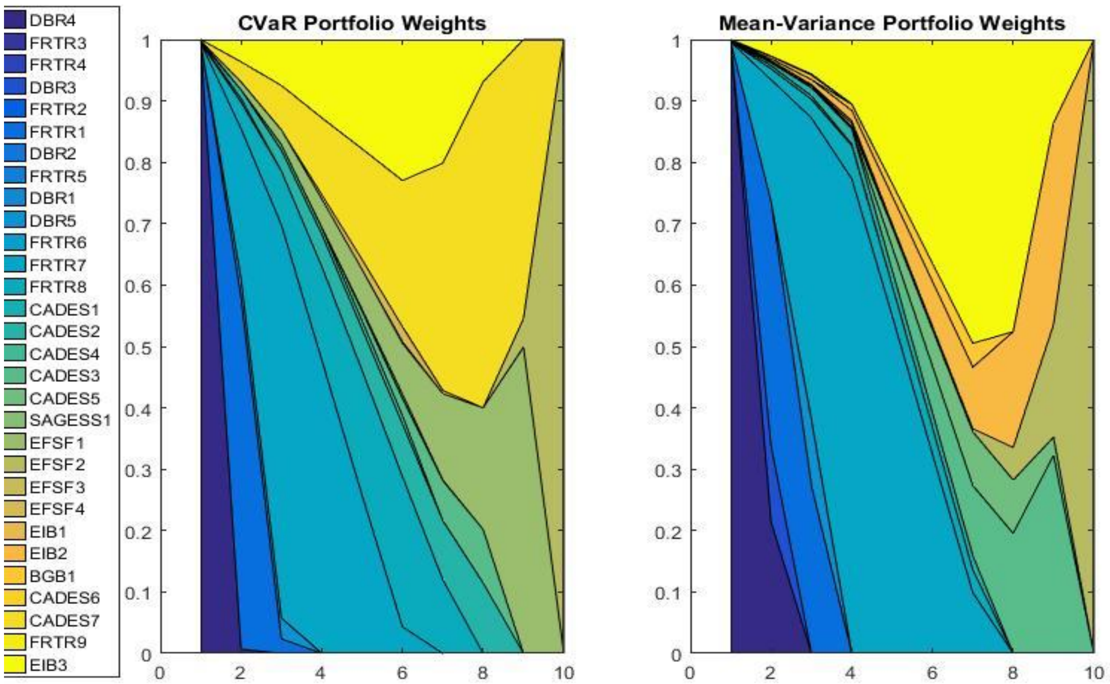

Figure 10 : Compare weights for two different approaches

We can see, that the two plots in Figure 10 show different distributions. Note also that the 11 first stocks are almost never in the optimal portfolio (independently of the return and/or the optimization method). For an investor it is more interesting to consider the weights where the returns are positive.

The Mean-Variance and the CVaR portfolio optimization on the level of 5% suggest that the last 19 stocks are included in the optimal portfolio but again in a completely different proportion. This results shows even clearer that we could make a different decision depending on the risk measure.

OPTIMAL ASSET WEIGHTS					
DBR4	3.92%	FRTR6	0.53%	EFSF2	5.26%
FRTR3	3.16%	FRTR7	4.31%	EFSF3	5.26%
FRTR4	2.21%	FRTR8	4.47%	EFSF4	5.26%
DBR3	3.92%	CADES1	1.32%	EIB1	1.33%
FRTR2	3.56%	CADES2	2.65%	EIB2	2.66%
FRTR1	4.45%	CADES4	5.30%	BGB1	5.36%
DBR2	4.11%	CADES3	1.32%	CADES6	2.11%
FRTR5	2.05%	CADES5	2.65%	CADES7	5.28%
DBR1	2.55%	SAGESS1	2.20%	FRTR9	3.20%
DBR5	2.63%	EFSF1	0.79%	EIB3	6.16%

Table 6 : List of asset weights from the minimization of CVaR

In this table, we shown the different weights of assets. The value of the CVaR is 0.0333 which is the expected loss on the investment's portfolio following the tail of the distribution 5%.

CONCLUSIONS

Sometimes, the financial institutions are regularly confronted to various risks in asset management. There is no exception for Central banks. In this study, we focus on Bank of Central African States where we have asked for the first time the question on this topic, through its portfolio of assets that can in the context of the management of their stocks to obtain the optimal portfolio. Notice that these stocks are exposed to financial risks, including market risk.

In this work, it was a question of measuring the level of loss that the BEAC asset management portfolio would suffer as a result of market fluctuations. The main objective of the study was to determine the portfolio's position relative to interest rate risk and to find an optimal portfolio by using a CVaR portfolio Optimization approach to solve a problem which we face. From this general objective came several specific objectives. At the end of these objectives were postulated two hypotheses which are all validated.

At the end of this work, operational risk management which is the selection and adjustment of risk management strategies and policies contributes to reduce potential losses to the portfolio. It is important for the bank to identify the risks inherent in new products and activities and to ensure that they are subject to appropriate procedures and controls in advance.

It is essential that the bank has a market risk measure system that covers the main sources of this risk and assesses the effects of changes in interest rates in ways that are consistent with the importance of their activities.

The bank must have an adequate system of internal controls over their risk management process and portfolio optimization. One of its fundamental components is to conduct a regular and independent review and assessment of the effectiveness of this system and to make the necessary changes and improvements. The results of these reviews should be made available to the relevant supervisory authorities.

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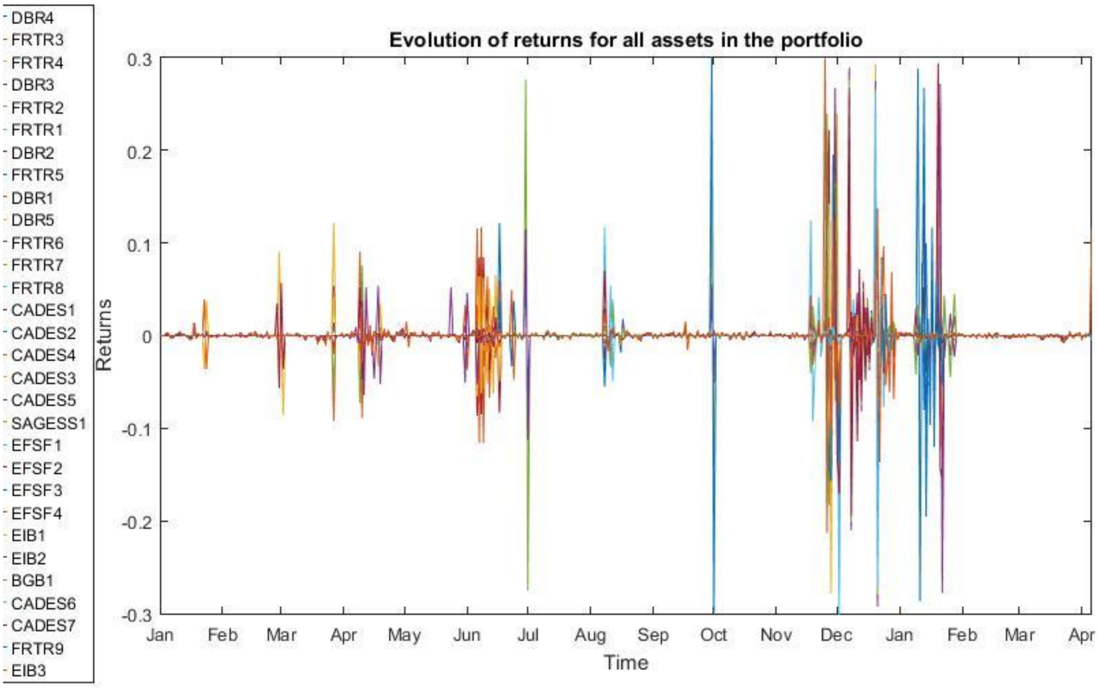
APPENDICES

Appendix 1: Exploration of the portfolio



Appendix 2: Normality test

Asset\Test	Shapiro-Wilk	Anderson-Darling	Lilliefors	Jarque-Bera
DBR4	> 0.0001	> 0.0001	> 0.0001	> 0.0001
FRTR3	> 0.0001	> 0.0001	> 0.0001	> 0.0001
FRTR4	> 0.0001	> 0.0001	> 0.0001	> 0.0001
DBR3	> 0.0001	> 0.0001	> 0.0001	> 0.0001
FRTR2	> 0.0001	> 0.0001	> 0.0001	> 0.0001
FRTR1	> 0.0001	> 0.0001	> 0.0001	> 0.0001
DBR2	> 0.0001	> 0.0001	> 0.0001	> 0.0001
FRTR5	> 0.0001	> 0.0001	> 0.0001	> 0.0001
DBR1	> 0.0001	> 0.0001	> 0.0001	> 0.0001
DBR5	> 0.0001	> 0.0001	> 0.0001	> 0.0001
FRTR6	> 0.0001	> 0.0001	> 0.0001	> 0.0001
FRTR7	> 0.0001	> 0.0001	> 0.0001	> 0.0001
FRTR8	> 0.0001	> 0.0001	> 0.0001	> 0.0001
CADES1	> 0.0001	> 0.0001	> 0.0001	> 0.0001
CADES2	> 0.0001	> 0.0001	> 0.0001	> 0.0001
CADES4	> 0.0001	> 0.0001	> 0.0001	> 0.0001
CADES3	> 0.0001	> 0.0001	> 0.0001	> 0.0001
CADES5	> 0.0001	> 0.0001	> 0.0001	> 0.0001
SAGESS1	> 0.0001	> 0.0001	> 0.0001	> 0.0001
EFSF1	> 0.0001	> 0.0001	> 0.0001	> 0.0001
EFSF2	> 0.0001	> 0.0001	> 0.0001	> 0.0001
EFSF3	> 0.0001	> 0.0001	> 0.0001	> 0.0001
EFSF4	> 0.0001	> 0.0001	> 0.0001	> 0.0001
EIB1	> 0.0001	> 0.0001	> 1.0001	> 0.0001
EIB2	> 0.0001	> 0.0001	> 0.0001	> 0.0001
BGB1	> 0.0001	>0.0001	> 0.0001	> 0.0001
CADES6	> 0.0001	>0.0001	> 0.0001	> 0.0001
CADES7	> 0.0001	> 0.0001	> 0.0001	> 0.0001
FRTR9	> 0.0001	> 0.0001	> 0.0001	> 0.0001
EIB3	> 0.0001	> 0.0001	> 0.0001	> 0.0001

Appendix 3: Matlab Code Step 1

```
%%
% This workflow is split up to 4 parts. They are:
% 1) Import data and Calculation of the log-returns
% 2) Find the optimal CVaR portfolio
% 3) Find the optimal Mean-Variance portfolio
% 4) Compare Mean-Variance with CVaR portfolios
clear; close all; clc;


%% Import data and calculation of matrix of returns
after = 2*22; % number of return
before = after; % service variable
before = before + 1;

pi_year = 2.5/100; % yearly expected returns the investors wishes
r_year = 1/100; % yearly returns of the riskless asset

pi_des = (1 + pi_year)^(1/252) - 1; % daily expected returns the investors
wishes
r_certo = (1 + r_year)^(1/252) - 1; % daily returns of the riskless asset

%data = dlmread('input1.txt'); %price import from txt
load AssetsPrices.csv;  %price import from csv
data = AssetsPrices(:,1:30);

dimension = size(data); % dimension of data
rig = dimension(1,1);
col = dimension(1,2);

for i=1:col; %cycle for return calculation
    rt = log( data(2:rig,i)./data(1:rig-1,i)); %returns of the i-th asset
    if i == 1;
       rend = rt;
    else
        rend = horzcat(rend, rt); %matrix of the returns
    end;
end;

minimum = min(min(rend)); %quantity for the graphs
maximum = max(max(rend)); %quantity for the graphs

Returns = rend(1:rig-before,:);
dim = size(Returns); %matrix dimensions for the means, variances and
covariances estimation

figure;
plot(Returns);
title('Evolution of returns for all assets in the portfolio');
xlabel('Time');
ylabel('Returns');
datetick;
axis([1 rig-before minimum maximum]);
legend('DBR4','FRTR3','FRTR4','DBR3','FRTR2','FRTR1','DBR2','FRTR5',...
    'DBR1','DBR5','FRTR6','FRTR7','FRTR8','CADES1','CADES2','CADES4',...
    'CADES3','CADES5','SAGESS1','EFSF1','EFSF2','EFSF3','EFSF4','EIB1',...
    'EIB2','BGB1','CADES6','CADES7','FRTR9','EIB3');
```

Appendix 4: Matlab Code Step 2

```
%% CVaR Portfolio
% Here we create the CVaR object and use the object's methods to pass in
% data and setup the CVaR problem. By setting the probality level to 0.95,
% we are choosing to minimize the mean loss in the 5% of portfolio
% returns with the highest losses.

%% Setup portfolio
% This way works because the calls to the |PortfolioCVaR| function are in
% this particular order. In this case, the call to initialize
%|AssetScenarios|
% provides the dimensions for the problem. If you were to do this step last,
% you would have to explicitly dimension the |LowerBound| property as
follows:
m = [ -0.000163716; -0.000143558; -0.000148968; -0.000139858; -0.000115316;
    -0.000122111; -0.000108615; -8.61657E-05; -0.000113894; -9.89779E-05;
    -6.34158E-05; 3.01676E-05; 0.000153802; 0.000149711; 0.000336354;
    0.000217672; 0.000510597; 0.00043779; 0.000437875; 0.000470383;
    0.000606886; 0.000463721; 0.00031653; 0.000248722; 0.000570018;
    0.000321222; 0.000334993; 0.000199583; 0.000187912; 0.000332839 ];
C = cov(Returns);

m = m/12;
C = C/12;

AssetScenarios = mvnrnd(m, C, 20000);

p = PortfolioCVaR('Scenarios', AssetScenarios, 'LowerBound', 0, ...
'Budget', 1, 'ProbabilityLe', 0.95);

p = PortfolioCVaR;
p = PortfolioCVaR(p, 'LowerBound', zeros(size(m)));
p = PortfolioCVaR(p, 'LowerBudget', 1, 'UpperBudget', 1);
p = setProbabilityLevel(p, 0.95);
p = setScenarios(p, AssetScenarios);

%% Plot the CVaR efficient frontier
figure;
plotFrontier(p);

%%
% If you did not specify the size of |LowerBound| but, instead, input a
% scalar argument, the |PortfolioCVaR| function assumes that you are defining
% a single-asset problem and produces an error at the call to set asset
% scenarios with 30 assets.

%%
[lb, ub, isbounded] = estimateBounds(p);

%% Mean-Variance Portfolio
% It's a good idea to look at a portfolio problem according two different
% risk measures. So let's create a mean-variance object, and find the 30
% efficient portfolios according to a mean-variance risk proxy. The
% workflow is very siilar to CVaR portfolio workflow, except that we use
% the estimateAssetMoments methods now, to estimate the mean and covariance
% for our returns data.
```

```
%% Setup portfolio

pmv = Portfolio('AssetMean', m, 'AssetCovar', C,...
'LowerBound', 0, 'UpperBudget',1, 'LowerBudget',1)

%% Plot the Mean-Variance efficient frontier
figure;
plotFrontier(pmv);

%% Compare CVaR and Mean-Variance Portfolios
%% Compare weights
% Another way to compare an efficient portfolios is to look at the weights
% for 30 portfolios. We'll visualize the weignts for the CVaR and the Mean-
Variance
% side-by-side using area plots. This allows us to see the mix of the
different assets
% in each of the 30 efficient portfolios.

pwgt = estimateFrontier(p);
pwmvgt = estimateFrontier(pmv);

figure;

subplot(1,2,1);
area(pwgt');
title('CVaR Portfolio Weights');

subplot(1,2,2);
area(pwmvgt');
title('Mean-Variance Portfolio Weights');

set(get(gcf,'Children'),'YLim',[0 1]);
legend(pmv.AssetList);
```

Matlab Code on the Calculation of Conditional Value at Risk and Optimal portfolio Weights

```
function [CVaR,w]=CVaRfunction(r, Er, b, ub, lb)
%CVaR - this function finds the Conditional Value at Risk of each asset in
%a given portfolio.
% INPUT:
% . r - matrix of returns
% . b - probability level (0.95<b<1)
% . lb - lowest bound for each stock weight (-1<lb<1)
% . ub - upper bound for each stock weight (-1<ub<1)
% always ub>lb
% . Er - target portfolio return (-1<Er<1)
% OUTPUT:
% . CVaR - is the Conditional Value at Risk for the portfolio
% . w - the optimal weights for each asset
[i, j]=size(r);
w0=[(1/j)*ones(1,j)];
VaR=quantile(r*w0',b);
w0=[w0 VaR];
% objective function
```

```
n=1:j;
Rfunction=@(w) w(j+1)+(1/i)*(1/(1-b))*sum(max(-w(n)*r(:,n)'-
w(j+1),0));
if isempty(Er),isempty(ub),isempty(lb)
% matrices
Aeq=[ ones(1,j) 0];
beq=[1];
% minimize the risk function
[w,CVaR]=fmincon(Rfunction,w0,[],[],Aeq,beq);
else
% matrices
A=[-mean(r) 0; -eye(j) zeros(j,1); eye(j) zeros(j,1)];
b=[-Er -lb*ones(1,j) ub*ones(1,j)]';
Aeq=[ ones(1,j) 0];
beq=[1];
% minimize the risk function considering the constraints on w and Er
[w,CVaR]=fmincon(Rfunction,w0,A,b,Aeq,beq,lb,ub,[]);
end
end
```